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# HYPERSURFACES WITH HARMONIC CURVATURE IN A SPACE OF CONSTANT CURVATURE

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## 1. Introduction and theorems

A Riemannian curvature tensor $R$ is said to be harmonic if it satisfies

$$\nabla_i R_{jk} - \nabla_j R_{ik} = 0$$

where $R_{ij}$ means the component of the Ricci tensor, i.e. $R_{jk} = R^i{}_{jik}$. If the Ricci tensor is parallel, the curvature is harmonic. However the converse is generally not true, [2]. Concerning this matter, we obtain some results in the case of hypersurfaces in a space of non-negative constant curvature. The purpose of this note is to prove the next theorems:

We denote the $k$-dimensional Euclidean space and the $k$-dimensional sphere of curvature $c$ by $E^k$ and $S^k(c)$ respectively.

THEOREM 1. *Let $M^n$ be a connected hypersurface with harmonic curvature, isometrically immersed in $E^{n+1}$ by an isometric immersion $\phi$ with constant mean curvature. We denote the second fundamental form by $h$.*

(i) *If $M^n$ is complete and trace $h^4$ is constant on $M^n$, then $\phi(M^n)$ is of the form $S^p \times E^{n-p}$, $0 \leqq p \leqq n$.*

(ii) *If $M^n$ is compact, then $\phi(M^n)$ is $S^n$.*

THEOREM 2. *Let $M^n$ be a connected hypersurface with harmonic curvature, isometrically immersed in $S^{n+1}(c)$ by an isometric immersion $\phi$ with constant mean curvature. If $M^n$ is complete and trace $h^4$ is constant on $M^n$, or if $M^n$ is compact, then $\phi(M^n)$ is of the form $S^p(r) \times S^{n-p}(s)$, $0 \leqq p \leqq n$, where $r = \alpha^2 + c$, $s = \beta^2 + c$, and $\alpha$ and $\beta$ satisfy $\alpha\beta + c = 0$ and $p\alpha + (n-p)\beta = \text{trace}\, h$.*

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## 2. The proof of theorems

We consider a hypersurface $M^n$ with harmonic curvature, isometrically

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