A. ARAI
KODAI MATH. J.
9 (1986), 161—164

ON UNIQUENESS THEOREM CONCERNING THE RENORMALIZED SCHWINGER-DYSON EQUATIONS OF FIRST ORDER

—A remark on the preceding paper by A. Inoue

BY ASAO ARAI

In the preceding paper [1]. Inoue considered some Schwinger-Dyson (SD) equations of first order requiring "renormalizations": he derived the renormalized SD-equation and constructed an explicit solution for each case. But the problem of the uniqueness is left open there.

Here we give an affirmative answer to that problem. The point of our method lies in a reduction of (SD)-equations to partial differential equations with finite numbers of variables of a certain type; it is essential that the order of the (SD)-equations is one.

For simplicity, we consider only the simplest one in [2]. The method can be easily extended to other cases. We shall carry over the notation in the preceding paper unless otherwise stated.

We first define a class of fundamental solutions of the d'Alembertian $\square$:

DEFINITION. Let G be a distribution on $\boldsymbol{R}^4\times\boldsymbol{R}^4$ satisfying

$$\square_x G(x, y)=\delta(x-y)$$

(G is called a fundamental solution of $\square$.) Then G is said to be in $\boldsymbol{F}$ if and only if it is a tempered distribution on $\boldsymbol{R}^4\times\boldsymbol{R}^4$ such that, for all $u\in S(\boldsymbol{R}^4)$,

$$(Gu)(0, t)=\lim_{\varepsilon\to 0}\langle\rho_\varepsilon, (Gu)(\cdot, t)\rangle$$

exists in $H^{-1}(\boldsymbol{R})$

Remark. The class $\boldsymbol{F}$ is rather "large".

We consider the following renormalized SD-equation for functionals $Z=Z(p, u)$ on $S(\boldsymbol{R})\times S(\boldsymbol{R}^4)$:

$$\left(\frac{d}{dt^2}+\omega_0^2-\frac{i\lambda^2}{4\pi}\left|\frac{d}{dt}\right|\right)\frac{\delta Z}{\delta p(t)}=\frac{i}{\hbar}p(t)Z-\frac{i\lambda}{\hbar}(Gu)(0, t)Z\,, \tag{1}$$

Received September 19, 1985