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KODAI MATH. J.
9 (1986), 84—116

A VARIFOLD SOLUTION TO THE NONLINEAR EQUATION OF MOTION OF A VIBRATING MEMBRANE

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§ 1. Introduction.

Let Ω be a bounded domain in $\boldsymbol{R}^n$ with the boundary $\partial\Omega$ which is a Lipschitz manifold. Then the equation of motion of a vibrating membrane is as follows:

$$D_t^2 u(t, x)-\sum_{j=1}^{n} D_j\{D_j u(t, x)(1+|Du(t, x)|^2)^{-1/2}\}=0, \qquad x\in\Omega, \tag{1.1}$$

where D_t denotes $\partial/\partial t$ and D_j denotes $\partial/\partial x_j$, $j=1, 2, \cdots, n$. The initial and the boundary conditions we shall consider are

$$u(0, x)=u_0(x), \qquad D_t u(0, x)=u_1(x), \tag{1.2}$$

$$u(t, x)=0 \quad \text{for} \quad x \quad \text{in} \quad \partial\Omega. \tag{1.3}$$

If $u_0(x)$ and $u_1(x)$ are sufficiently smooth, there exists a unique genuine solution of (1.1), (1.2) and (1.3) for a short time interval. (cf. Kato [9] and Shibata-Tsutsumi [10]). On the other hand, existence global in time of even a weak solution is not proved in the case $n>1$.

The purpose of the present paper is to treat the above equation by virtue of the theory of varifolds introduced by Almgren Jr. [2]. A varifold is a generalization of the notion of a function and was successfully used in the direct approach of the Plateau's problem. We shall define a generalized solution of the equation (1.1) in terms of varifolds, which we call the varifold solution. And we shall prove existence, global in time, of a varifold solution of (1.1), (1.2) and (1.3). Thus this paper is closely related with the works of Tartar [11], [12] and that of DiPerna [5].

Although a varifold solution is quite a weak notion, it satisfies a generalization of the Hamilton's principle:

$$\delta\int_0^T dt\int_\Omega\left\{\frac{1}{2}|D_t u(t, x)|^2-(1+|Du(t, x)|^2)^{1/2}\right\}dx=0 \tag{1.4}$$

under appropriate assumptions.

Before introducing a varifold solution, we shall formulate, in § 2, the notion

Received April 1, 1985