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SOME RESULTS IN GEOMETRY OF HYPERSURFACES

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0. Introduction.

In this paper we get several theorems about hypersurfaces in space forms.

In section 1, we show that if $x: M^n \to E^{n+1}$ is an isometric immersion of an n-dimensional complete non-compact Riemannian manifold whose sectional curvatures are greater than or equal to 0, then $x(M)$ is unbounded in E^{n+1}. We can prove this using Sacksteder theorem [12] which states that under the above condition $x(M)$ is the boundary of a convex body in E^{n+1}. But his proof is rather long and his theorem is more than what we need. do. Carmo and Lima [3] gave an independent proof of Sacksteder theorem, but it is also long. So we give a direct and easy proof using so-called Beltrami maps which are defined in do. Carmo and Warner [4].

In section 2, we show that if $x: M^n \to S^{n+1}(1)$ is an isometric immersion of an n-dimensional complete Riemannian manifold whose sectional curvatures are less than or equal to 1 and n is greater than 3, then $x(M)$ is totally geodesic. Ferus almost proved this result in [6], [7]. We consider higher codimensional cases.

All manifolds we consider in this paper are class C^∞, connected and have dimensions greater than or equal to 2. All immersions and vector fields are C^∞.

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1. Unboundedness of hypersurfaces.

The Beltrami maps are defined in M. do Carmo and F. Warner [2], and their properties are discussed fully.

Let $\nu \in S^{n+1}(1)$ $(\subset E^{n+2})$, and let H_ν denote the open hemisphere of $S^{n+1}(1)$ centered at ν. The Beltrami map β_ν is the diffeomorphism of H_ν onto the hyperplane $S_\nu \subset E^{n+2}$ tangent to $S^{n+1}(1)$ at ν obtained by central projection. We consider S_ν to be equipped with the canonical Riemannian structure induced from E^{n+2}. β_ν map great spheres of the sphere onto planes of S_ν, and vice versa. We call this Beltrami map as spherical Beltrami map.

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