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KODAI MATH. J.
9 (1986), 68—76

PROBABILISTIC CHARACTERIZATION OF CERTAIN BANACH SPACES

Dedicated to Professor Hisaharu Umegaki on his 60th birthday

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1. Introduction.

Let $(\Omega, \mathcal{A}, P)$ be a probability measure space, X a real separable Banach space and X^* its topological dual space. In this paper we look into the relation between the convergence of a sequence $(\xi_n)_{n\geq 1}$ of random variables with values in X (for short, random elements) and the convergence of the associated sequence $(\langle\xi_n, f\rangle)_{n\geq 1}$, $f\in X^*$, of real random variables. In [8], when X is a Hilbert space, the author has treated this problem for mean square convergence with the aid of covariance operators or more specifically, with the aid of the relative compactness of covariance operators with respect to the *trace norm.* In Chevet-Chobanjan-Linde-Tarieladze [1, 10] they showed that covariance operators on Banach spaces introduced in Vakhania [14] are nuclear operators which were defined by Grothendieck [6]. This leads us to consider the following

PROPOSITION (A) *ξ_n converges in $L_2(\Omega; X)$ if and only if the following two conditions are satisfied:*

(a) *For each $f\in X^*$, $\langle\xi_n, f\rangle$ converges in $L_2(\Omega; \boldsymbol{R})$, where $\boldsymbol{R}$ is the real line.*

(b) *The set $\{R_{\xi_n}\}$ of covariance operators is relatively compact with respect to the nuclear norm.*

This Proposition (A), however, is not valid in general. The purpose of this paper is to find necessary and sufficient conditions on the structure of Banach spaces in order that the Proposition (A) holds.

In section 2 we shall give some definitions and preliminary results. In Section 3 we shall show that the Proposition (A) is not valid unless X is isomorphic to a Hilbert space. This result will be proved by an isomorphic characterization of Hilbert spaces which was obtained by Kwapień [9]. In Section 4 we shall also prove that the Proposition (A) holds only for Gaussian random elements if and only if X is of type 2.

The author thanks Professor Hisaharu Umegaki for his valuable advice and encouragement. He also thanks Professor Fumio Hiai who carefully read the

Received March 19, 1985