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AN OPERATOR THEORETICAL CHARACTERIZATION OF ε-ENTROPY IN GAUSSIAN PROCESSES

Dedicated to Professor Hisaharu Umegaki on his sixtieth birthday

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1. Introduction.

In this paper, we shall treat ε-entropy of compact operators on a Hilbert space and that of measurable stochastic processes. Especially, using the concept of ε-entropy, we study a relation between Gaussian processes and integral kernel operators. In section 2, we shall explain the definition of ε-entropy in compact operators due to Prosser [9] and that in measurable stochastic processes due to Kolmogorov [5]. In section 3, we shall treat a mean continuous Gaussian process $\xi=\{\xi(t):0\leqq t\leqq 1\}$. Using the covariance function $K(s, t)$ induced by ξ, we can construct the integral kernel operator T on $L^2[0, 1]$, which is a trace class operator. Denote $S(T, \varepsilon)$ and $H(\xi, \varepsilon)$ the ε-entropies of T and ξ, respectively. We characterize the ε-entropy $H(\xi, \varepsilon)$ by the sequence:

$$\{S(T^k, \varepsilon^k): k=1, 2, \cdots\}.$$

In section 4, we shall consider the orders of growth of $H(\xi, \varepsilon)$ and $S(T, \varepsilon)$. Then, applying the result of Section 3, we estimate an upper bound of the order of growth of $H(\xi, \varepsilon)$.

Unless stated otherwise, throughout this paper, the letters R, Z and N denote the set of real numbers, the set of integers and the set of natural numbers, respectively.

2. Preliminaries.

In this section, we shall introduce several notations and definitions throughout this paper. Denote by $\mathcal{H}$ a Hilbert space whose inner product is $\langle\cdot, \cdot\rangle$. $B(x, \varepsilon)$ means an open ball having the radius $\varepsilon>0$ and the center $x\in\mathcal{H}$. Especially, denote by $\mathcal{U}$ the closed unit sphere in $\mathcal{H}$.

By an *ε-covering* of a subset F in $\mathcal{H}$, we mean a family of open balls with centers in $\mathcal{H}$ and radiuses ε, whose union covers F. By an *ε-packing* of F, we mean a family of open balls with centers in F and radiuses ε, whose pairwise

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