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ON THE GAUSS MAP OF MINIMAL SURFACES IMMERSED IN R^n

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1. Introduction.

The Gauss map of a minimal surface M in R^n can be considered as a holomorphic mapping from M to the complex quadric Q_{n-2} in the complex projective space CP^{n-1} with the Fubini-Study metric of constant curvature 2. This paper is devoted to the question, "If a minimal surface M in R^n has a constant curvature $\hat{K}$ in its Gaussian image, what values of $\hat{K}$ can be possible?".

This question comes from Ricci's classical theorem;

There exists a minimal surface in R^3 which is isometric with M iff (M, ds^2) satisfies Ricci condition:

(i) Gaussian curvature K of M is negative,

(ii) the new metric $d\tilde{s}^2=\sqrt{-K}ds^2$ is flat on M.

The condition (ii) is known to be equivalent to the condition "$\hat{K}\equiv 1$". (see Lawson [2])

Concerning the question, the following are well-known;

(a) If $\hat{K}\equiv 1$, then M must lie fully in R^3 or R^6. And all the minimal surfaces isometric to M make a two parameter family. (Lawson [2])

(b) Minimal surfaces in R^4 which have constant curvature $\hat{K}$ in their Gaussian images are classified as follows;

i. $\hat{K}\equiv 1$, and M lies in some affine R^3,

ii. $\hat{K}\equiv 2$, and M is a holomorphic curve in C^2.

Here C^2 means R^4 with some orthogonal complex structure. (Osserman-Hoffman [5])

(c) And in R^5,

i. $\hat{K}\equiv 1$ or 2, and M lies in R^4 (these are the cases (b).)

ii. $\hat{K}\equiv 1/2$, and the Gaussian image of M can be represented locally as;

$$1/2(1-w^4,\ i+iw^4,\ 2w+2w^3,\ 2iw-2iw^3,\ 2\sqrt{3}iw^2)$$

(Masal'tsev [4])

To get these results, Calabi's theorem [1] plays the main role. Using the

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