H. UEDA
KODAI MATH. J.
9 (1986), 33—43

ASYMPTOTIC BEHAVIOR OF CERTAIN SMALL SUBHARMONIC FUNCTIONS IN $\{\mathrm{Re}\ z>0\}$

BY HIDEHARU UEDA

1. Notation.

Let $\boldsymbol{C}$ be the complex plane. If $u(z)$ is subharmonic in a region $\Omega\subset\boldsymbol{C}$, we put

$$M(r,\ u)=\sup_{\substack{|z|=r\\ z\in\Omega}} u(z)\,.$$

Let $\partial\Omega$ be the boundary of Ω. If $\zeta\in\partial\Omega$ and $u(z)$ is subharmonic in Ω, we define

$$u(\zeta)=\limsup_{\substack{z\to\zeta\\ z\in\Omega}} u(z)\,.$$

2. Statement of Theorem.

In our previous paper [4], the following result is proved.

THEOREM A. *Let $u(z)$ be subharmonic in $\{\mathrm{Re}\, z>0\}$. If $u(z)$ satisfies the conditions*

$$u(0)<\infty \tag{2.1}$$

and

$$u(iy)\leqq M^{+}(|y|,\ u)-\pi^{2}\sigma \quad (-\infty<y<+\infty,\ y\neq 0\,;\ \sigma:\ a\ positive\ constant), \tag{2.2}$$

then either $u(z)\leqq-\pi^{2}\sigma$ in $\{\mathrm{Re}\, z>0\}$ or

$$\lim_{r\to\infty}\frac{M(r,\ u)-4\sigma(\log r)^{2}}{\log r}=\alpha \qquad (-\infty<\alpha\leqq+\infty)\,. \tag{2.3}$$

It seems to be interesting to investigate the asymptotic behavior of the subharmonic functions in $\{\mathrm{Re}\, z>0\}$ satisfying the conditions (2.1), (2.2) and (2.3) with a finite number α. In this note we prove

THEOREM. *Suppose that $u(z)$ is subharmonic in $\{\mathrm{Re}\, z>0\}$ and satisfies* (2.1), (2.2) *and* (2.3) (*where α is finite*) *with a suitable positive number σ. Suppose further that for any $r>0$ there exists z_r such that*

Received March 5, 1985