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THE GAUSS IMAGE OF FLAT SURFACES IN R^4

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Let M be a surface of zero Gaussian curvature in $\boldsymbol{R}^4$ which has flat normal connection. Let $G_{2,4}$ denote the Grassmann manifold consisting of oriented 2-dimensional linear subspaces of $\boldsymbol{R}^4$. The Gauss map $G: M \to G_{2,4}$ is defined by assigning each point of M to the tangent plane of M at the point.

In this paper we study the structure of the image of M by the Gauss map. In [4], C. Thas showed that the Gauss image of a surface of zero Gaussian curvature in $\boldsymbol{R}^4$ which has flat normal connection is flat. Theorem 1 gives a further information on the structure of the Gauss image. Namely, under the identification $G_{2,4}=S^2\left(\frac{1}{\sqrt{2}}\right)\times S^2\left(\frac{1}{\sqrt{2}}\right)$, the Gauss image of M is the Riemannian product of two curves, one in the first factor of $S^2\times S^2$ and one in the second factor. We compute the geodesic curvatures of those curves and show that if those curves are totally geodesic, then M is the Riemannian product of two plane curves.

In § 2, we give some local formulas for principal curvatures and show that if certain functions defined from principal curvatures vanish everywhere, then the surface is the Riemannian product of two plane curves. In § 3, we look at $G_{2,4}$ and give some basic formulas. In § 4, we prove our theorems for the geometry of the Gauss image of M.

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1. Preliminaries.

Let M be a connected n-dimensional C^∞ Riemannian manifold and let $\psi: M \to \boldsymbol{R}^N$ be an isometric immersion of M into an N-dimensional Euclidean space $\boldsymbol{R}^N$. Let D and $\bar{D}$ denote the covariant differentiations of M and $\boldsymbol{R}^N$ respectively. Let X, Y be tangent vector fields on M. Then

$$\bar{D}_X Y = D_X Y + B(X, Y) \tag{1.1}$$

where $B(X, Y)$ is the normal component of $\bar{D}_X Y$.

Let ξ be a normal vector field on M. We write

$$\bar{D}_X \xi = -A_\xi X + D_X^\perp \xi \tag{1.2}$$

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