S. TANNO AND K, MASUDA
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A CHARACTERIZATION OF THE PRODUCT OF TWO 3-SPHERES BY THE SPECTRUM

BY SHUKICHI TANNO AND KAZUO MASUDA

§1. Introduction.

Let (M, g) be a compact Riemannian manifold. By $\mathrm{Spec}(M, g)$ we denote the spectrum of the Laplacian acting on functions on (M, g). Let $S^m(c)$ be the m-sphere of constant curvature c.

For $m \leqq 6$, $S^m(c)$ is characterized by the spectrum (Berger [1], Tanno [5]); that is, $\mathrm{Spec}(M, g) = \mathrm{Spec}\, S^m(c)$ implies that (M, g) is isometric to $S^m(c)$.

For $m \geqq 7$, it is an open question if $S^m(c)$ is characterized by the spectrum. As for partial answers see [6].

In this paper we obtain the following theorem on product Riemannian manifolds.

THEOREM A. *Let (M, g) and (M', g') be 3-dimensional compact Riemannian manifolds. Assume that*

$$\mathrm{Spec}\,[(M, g) \times (M', g')] = \mathrm{Spec}\,[S^3(c) \times S^3(c')].$$

Then, (M, g) and (M', g') are of constant curvature K and K', respectively, and $K + K' = c + c'$.

Furthermore, if the sectional curvatures K and K' are positive, then (M, g) is isometric to $S^3(c)$ (or $S^3(c')$) and (M', g') is isometric to $S^3(c')$ (or $S^3(c)$, resp.).

Let $CP^n(H)$ be the n-dimensional complex projective space of constant holomorphic sectional curvature H. Corresponding to Theorem A we get

THEOREM B. *Let (M, g, J) and (M', g', J') be (complex) 3-dimensional compact Kählerian manifolds. Assume that*

$$\mathrm{Spec}\,[(M, g, J) \times (M', g', J')] = \mathrm{Spec}\,[CP^3(H) \times CP^3(H')].$$

Then, (M, g, J) is holomorphically isometric to $CP^3(H)$ (or $CP^3(H')$) and (M', g', J') is holomorphically isometric to $CP^3(H')$ (or $CP^3(H)$, resp.).

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