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A CERTAIN PROPERTY OF GEODESICS OF THE FAMILY OF RIEMANNIAN MANIFOLDS O_n^2 (VII)

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§0. Introduction.

This is exactly a continuation of Part (VI) ([16]) with the same title written by the present author which proved the following conjecture is true for $9.7 \leqq n \leqq 16$. On the methods used in it, the lower bound 9.7 of this effective interval is near the crucial values from the argument in it. We shall show that this conjecture is also true for $5 \leqq n \leqq 9.7$ in the present paper by improving them and some new ideas. As the previous one we shall use the numerical data obtained by computors in the verification. We shall also use the same notation in the previous ones, Parts (I)∼(VI).

The period T of any non-trivial solution $x(t)$ of the non-linear differential equation of order 2:

$$\text{(E)} \qquad nx(1-x^2)\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2+(1-x^2)(nx^2-1)=0$$

with a constant $n>1$ such that $x^2+x'^2<1$ is given by the integral:

$$\text{(0.1)} \qquad T=\sqrt{nc}\int_{x_0}^{x_1}\frac{dx}{x\sqrt{(n-x)\{x(n-x)^{n-1}-c\}}},$$

where $x_0=n\{\min x(t)\}^2$, $x_1=n\{\max x(t)\}^2$, $0<x_0<1<x_1<n$ and $c=x_0(n-x_0)^{n-1}=x_1(n-x_1)^{n-1}$.

CONJECTURE C. The period T as function of $\tau=(x_1-1)/(n-1)$ and n is monotone decreasing with respect to n (>2) for any fixed τ $(0<\tau<1)$.

Here the author thanks heartily Professor Naoto Abe for his cooperation in the numerical computations by computors.

§1. The fundamental principle to attain the purpose.

Setting $T=\Omega(\tau, n)$, we have the formulas

$$\text{(1.1)} \qquad \frac{\partial\Omega(\tau, n)}{\partial n}=-\frac{\sqrt{c}}{2b^2n\sqrt{n}}\int_{x_0}^{1}\frac{(1-x)\sqrt{x(n-x)^{n-1}-c}\,V(x, x_1)dx}{x^2(n-x)^n}$$

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