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# FINITE TYPE SUBMANIFOLDS IN PSEUDO-EUCLIDEAN SPACES AND APPLICATIONS

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## §1. Introduction.

Let $E_s^m$ be the $m$-dimensional pseudo-Euclidean space with (flat) pseudo-Riemannian metric of signature $(s, m-s)$. And let $M$ be a compact space-like submanifold of $E_s^m$. By using the induced Riemannian structure on $M$, we can define two well-defined numbers $p$ and $q$ associated with the submanifold $M$ in $E_s^m$. Here $p$ is a positive integer and $q$ is either $+\infty$ or an integer $\geqq p$. The pair $[p, q]$ is called the *order of the submanifold* $M$ (cf. [1]). The submanifold $M$ is said to be of *finite type* if $q$ is finite. Otherwise, $M$ is said to be of *infinite type*. The submanifold $M$ is of finite type if and only if there is a non-trivial polynomial $P$ such $P(\Delta)H=0$; where $\Delta$ is the Laplacian $\Delta$ on $M$ and $H$ the mean curvature vector of $M$ in $E_s^m$.

In this paper, we will give some general results for finite type submanifolds in the pseudo-Euclidean space $E_s^m$. By applying these results, we will prove the following. (1) There exist no compact space-like hypersurfaces with constant mean curvature and constant scalar curvature in the anti-de Sitter space-time; (2) Every compact hypersurface with constant mean curvature and constant scalar curvature in a hyperbolic space is a small hypersphere; and (3) If $M$ is a compact space-like hypersurface of the de Sitter space-time, then $M$ has non-zero constant mean curvature and constant scalar curvature when and only when $M$ is mass-symmetric and of 2-type in the Lorentz-Minkowski world.

For the general knowledge on Finite-Type Submanifolds in Euclidean spaces, see [1, 2]. And for the general knowledge on Relativity, see for instance [3, 4].

## §2. Preliminaries.

Let $E_s^m$ be the $m$-dimensional pseudo-Euclidean space with metric tensor given by

$$g_0=-\sum_{i=1}^{s} dx_i^2+\sum_{j=s+1}^{m} dx_j^2, \tag{2.1}$$

where $(x_1, \cdots, x_m)$ is a rectangular coordinate system of $E_s^m$. $(E_s^m, g_0)$ is a flat pseudo-Riemannian manifold of signature $(s, m-s)$.

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