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ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF A CERTAIN SECOND ORDER ORDINARY DIFFERENTIAL EQUATION

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§ 1. Introduction.

Consider the following ordinary differential operator of second order:

$$P=\partial_t^2+k^2(t^2+y^2)+kc. \tag{1.1}$$

Here k belongs to the set $\boldsymbol{R}_+$ of the positive numbers, c to the complex number field $\boldsymbol{C}$, and (t, y) lies in the Euclid plane $\boldsymbol{R}^2$.

The purpose of the present article is to give a fundamental pair of the solutions to the equation:

$$Pu=0 \tag{1.2}$$

with detailed asymptotic properties as $k\to+\infty$. The novelty we claim here is its derivation as we will roughly sketch immediately after the statement of our Main Theorem (Theorem 1.1) below. Our asymptotic expansions are in fact different from usually given ones (see Nishimoto [5]). We expect that our results will be extended to partial differential operators such as

$$\partial_t^2-\partial_x^2+k^2(t^2+x^2+y^2)+kc.$$

Details on the latter case will be discussed elsewhere.

Now we explain what will be required in our formulation of asymptotics. Let

$$T_\varepsilon(t, y)=\varepsilon t+\sqrt{t^2+y^2}, \qquad \varepsilon\in\{+, -\}, \tag{1.3}$$

and denote by D_ε the set of (t, y, k) such that $k>0$, $T_\varepsilon(t, y)>0$, y running on the real line $\boldsymbol{R}$. Thus, D_ε is the portion of the half space $\boldsymbol{R}_+^3=\{(t, y, k); k>0\}$ obtained by deleting the quarter plane $\{(t, y, k); \varepsilon t\leqq 0, y=0, k>0\}$. D_ε and the quarter space $\boldsymbol{R}_+\times\boldsymbol{R}\times\boldsymbol{R}_+$ are diffeomorphic by the bijection:

$$\Phi_\varepsilon(t, y, k)=(T_\varepsilon(t, y), y, k), \qquad (t, y, k)\in D_\varepsilon.$$

We will consider everything in the Fréchet space $\mathcal{E}(D_\varepsilon)$ of the infinitely differen-

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