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# A REMARK ON ALGEBRAIC GROUPS ATTACHED TO HODGE-TATE MODULES

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Let $K$ be a local field of characteristic 0 with the algebraically closed residue field of characteristic $p>0$. We consider a semi-simple Hodge-Tate module $V$ over $K$ with $V_C=\boldsymbol{C}\otimes_{\boldsymbol{Q}_p}V=V_C(0)\oplus V_C(1)$, $n_0=\dim V_C(0)\geqq 1$ and $n_1=\dim V_C(1)\geqq 1$. Let $H_V$ be the algebraic group attached to $V$, $H_V{}^o$ be the neutral component of $H_V$ and $\mathfrak{g}_V$ be their Lie algebra.

In [5] Serre has proved that $H_V=\boldsymbol{GL}_V$ if $n_0$ and $n_1$ are relatively prime and if $V$ is an absolutely simple $\mathfrak{g}_V$-module. He also remarked the possibility of determination of the structure of $H_V{}^o$ for other cases. For example, in [6] he has proved that all the irreducible components of the root system of $H_V{}^o$ are of type A, B, C or D and furthermore are of type A if $V$ is irreducible of odd dimension.

In this paper we prove that all the irreducible components of the root system of $H_V{}^o$ are of type A if $n_0\neq n_1$ and if $V$ is an absolutely simple $\mathfrak{g}_V$-module.

## § 1. Irreducible components of the root system.

In this section we use the following notations (cf. [6], § 3).

$\boldsymbol{Q}$=the field of rational numbers.

$E$=a field of characteristic 0.

$\boldsymbol{G}_m$=the one-dimensional multiplicative algebraic group over $E$.

$M$=a connected reductive algebraic group defined over $E$.

$E'$=a finite Galois extension of $E$ over which $M$ splits.

$\Gamma$=the Galois group of $E'/E$.

$C$=an algebraically closed field containing $E'$.

$T$=a splitting maximal torus of $M_{/E'}$, where $M_{/E'}$ denotes the scalar extension to $E'$ of $M$.

$X$=the character group of $T$.

$Y$=the group of the one-parameter subgroups of $T$.

$X_{\boldsymbol{Q}}=\boldsymbol{Q}\otimes X$.

$Y_{\boldsymbol{Q}}=\boldsymbol{Q}\otimes Y$.

$\langle x, y\rangle (x\in X_{\boldsymbol{Q}}, y\in Y_{\boldsymbol{Q}})$=the canonical bilinear form on $X_{\boldsymbol{Q}}\times Y_{\boldsymbol{Q}}$.

$R$=the root system of $M_{/E'}$ relative to $T$.

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