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HOMOTOPY TYPES OF CONNECTED SUMS OF SPHERICAL FIBRE SPACES OVER SPHERES

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§ 1. Introduction.

In the homotopy classification problems of highly-connected Poincaré complexes, the connected sums of spherical fibre spaces over spheres appear frequently. Especially, the manifolds with certain tangential and homotopy properties come to connected sums of sphere bundles over spheres (for example, [12]).

On the other hand, I.M. James and J.H.C. Whitehead classified homotopy types of the total space of sphere bundles over spheres in [3] and [4], and their results were extended to the case of spherical fibre spaces over spheres by S. Sasao in [7].

Motivated by those, H. Ishimoto classified connected sums of sphere bundles over spheres up to homotopy types in [2]. Then the purpose of this paper is to extend Ishimoto's results in [2] to the case of connected sums of spherical fibre spaces over spheres with cross-sections, which is also a generalization of Sasao's Theorem given in [7].

Let G_{n+1} be the space of maps of a n-sphere S^n to itself with degree 1 and F_n be the subspace of G_{n+1} consisting of maps preserving the base point $s_0={}^t(1, 0, 0, \cdots, 0)\in S^n$. Let X be a total space of an orientable n-spherical fibre space over a $(n+k+1)$-sphere which admits a cross-section, and we denote its characteristic element by $\chi(X)\in\pi_{n+k}(G_{n+1})$. Since X has a cross-section, we may suppose

$$(1.1) \qquad \chi(X)=j_{n*}(\gamma) \qquad \text{for some element} \quad \gamma\in\pi_{n+k}(F_n),$$

where

$$j_n : F_n \longrightarrow G_{n+1} \qquad \text{denotes the inclusion map.}$$

Let

$$\lambda : \pi_{n+k}(F_n) \xrightarrow{\cong} \pi_{n+k}(G_{n+1})$$

be the isomorphism defined by B. Steer in [10], and two maps

$$i' : SO_{n+1} \longrightarrow G_{n+1}$$

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