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ON THE BOUNDARY BEHAVIOR OF FUNCTIONS FOR WHICH THE RIEMANN IMAGE HAS FINITE SPHERICAL AREA

Dedicated to Mitsuru Ozawa on the Occasion of his Sixtieth Birthday

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§1. In 1912 Montel [10] proved a result which may be formulated as follows. Let $f(z)$, $z=x+iy$, be regular and bounded in a half-strip: $a<x<b$, $y>0$ and let $f(\xi+iy)$ have a limit as y tends to infinity for a fixed ξ, $a<\xi<b$. Then $f(x+iy)$ has the same limit as y tends to infinity for each x in (a, b), uniformly on any closed subinterval of (a, b). In 1915 Lindelöf [9] showed that the same result follows from the assumption that the limit exists on a more general path. Hardy, Ingham and Pólya [5] considered in the formulation of Montel the problem for $|f(x+iy)|$ rather than $f(x+iy)$ and found that the existence of the limit for the modulus for one ξ was not enough but that if $\lim_{y\to\infty}|f(x+iy)|$ existed for $x=\alpha$, β with $a<\alpha<\beta<b$ and $\beta-\alpha<(b-a)/2$ conclusions analogous to Montel's are obtained. They gave also some extensions and embellishments were made by Miss Cartwright [2], Hayman [6] and Bowen [1]. We will study a similar problem for the class of functions such that the Riemann image given by the mapping has finite spherical area and will find that then the existence of the limit for the modulus on one line implies a situation analogous to Montel's result. It turns out that results of this sort can be most appropriately formulated in terms of cluster sets. Also we can substantially weaken the requirements on the set of approach. Finally even in the family of bounded functions the cluster set interpretation provides new insights.

§2. Our results are most conveniently stated in terms of a half-strip for approach to its boundary point at infinity so we indicate briefly our terminology.

DEFINITION 1. Let S denote the half-strip $a<x<b$, $y>0$. Let T be a subset of S, T_λ its subset on which $y\geqq\lambda$. For f defined on S the cluster set $C(f, T, \sigma)$ of f on T at σ, the boundary point of S at the point at infinity, is defined to be $\bigcap_{\lambda>0}\mathrm{Cl}\, f(T_\lambda)$ where Cl denotes closure on the sphere.

DEFINITION 2. $\mathcal{F}(S)$ denotes the family of functions meromorphic on S for

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