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KODAI MATH. J.
8 (1985), 307—316

A CERTAIN SPACE-TIME METRIC AND SMOOTH GENERAL CONNECTIONS

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Introduction.

For a manifold M with a general connection Γ we say a connected subset A is a black hole, if it has a neighborhood U such that if any one going on along a geodesic enters U, then he will be finally swallowed in A. The present author gave a way in [8] by which we can construct a general connection Γ for any Riemannian manifold (M, g) and any point p of M such that Γ has p as a black hole and has the same system of geodesics as the one of (M, g) outside of a neighborhood.

In the theory of general relativity, the Eddington-Finkelstein metric g is given by

$$d\tau^2 = -\left(1-\frac{2m}{r}\right)dt^2 + 2dtdr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{1}$$

where (r, θ, φ) are the polar coordinates of the space R^3 with the coordinates (x_1, x_2, x_3) as

$$r=\sqrt{\Sigma x_i^2}, \quad x_1 = r\sin\theta\cos\varphi, \quad x_2 = r\sin\theta\sin\varphi, \quad x_3 = r\cos\theta .$$

As is well known, the curve $r=0$ in the space-time is a black hole as is mentioned above, even though the metric (1) loses the meaning along this curve, (1) is locally equivalent to the Schwarzschild metric

$$d\tau^2 = -\frac{r-2m}{r}dt^2 + \frac{r}{r-2m}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{2}$$

through the change of time t in (2) to $t-r-\log|r-2m|^{2m}$. (2) loses its meaning where $r=0$ and $r=2m$ but (1) is everywhere regular except $r=0$.

Now, we denote the affine connection made by the Christoffel symbols from the space-time metric (1) by Γ_g. Taking a tensor field P of type (1, 1), consider the general connection $\Gamma = P\Gamma_g$. Then, any geodesic of Γ_g is also a geodesic with respect to Γ. Conversely any geodesic of Γ is also a geodesic with respect to Γ_g, where P is an isomorphism on the tangent space of $R\times(R^3-\{0\})$. We consider a problem: Taking P suitably, is it possible $\Gamma = P\Gamma_g$ to extend smoothly

Received September 18, 1984