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KODAI MATH. J.
8 (1985), 296—306

ON FINITE MODIFICATIONS OF ALGEBROID SURFACES

Dedicated to Professor Yukio Kusunoki on his 60th birthday

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§ 1. Introduction.

Let R be an open Riemann surface, $\mathfrak{M}(R)$ the family of non-constant meromorphic functions on R and $P(f)$ the number of values which are not taken by $f\in\mathfrak{M}(R)$. We denote by $P(R)$ the Picard constant of R defined by

$$P(R)=\sup\{P(f)\,;\ f\in\mathfrak{M}(R)\}.$$

In general we have $P(R)\geqq 2$. The significant meaning of this Picard constant lies in the following fact:

THEOREM A (Ozawa [9]). *If $P(R)<P(S)$ for another Riemann surface S, then there is no non-trivial analytic mapping of R into S.*

From now on we shall confine ourselves to finitely sheeted covering algebroid surfaces defined as proper existence domains of algebroid functions. From the theory of algebroid functions we have $P(R_n)\leqq 2n$ for an n-sheeted algebroid surface R_n. An n-sheeted algebroid surface R_n is called regularly branched when it has no branched point other than those of order $n-1$.

Let $\mathfrak{E}_n$ be the family of entire functions having an infinite number of zeros whose orders are coprime to n and $\mathfrak{E}_n^*$ the subfamily of $\mathfrak{E}_n$ consisting of entire functions orders of all zeros of which are less than n.

We denote by R_n and $\tilde{R}_n$ two algebroid surfaces defined by $y^n=G(z)$ and $y^n=\tilde{G}(z)$, respectively, where $G(z)$ and $\tilde{G}(z)$ belong to $\mathfrak{E}_n^*$. If $G(z)$ has the same zeros with the same multiplicity as $\tilde{G}(z)$ in $|z|\geqq r_0$ for a suitable positive number r_0 and has at least one distinct zero with the multiplicity from $\tilde{G}(z)$ in $|z|<r_0$, then we call $\tilde{R}_n$ a finite modification of R_n (cf. Ozawa [11]).

We now consider two n-sheeted, regularly branched algebroid surfaces R_n and $\tilde{R}_n$ and two m-sheeted, regularly branched algebroid surfaces S_m and $\tilde{S}_m$. Suppose that $P(R_n)=2n$, $P(S_m)=2m$ and $\tilde{R}_n$ and $\tilde{S}_m$ are finite modifications of R_n and S_m, respectively. In our previous paper [8] we had a perfect condition for the existence of analytic mappings of R_n into S_m and investigated the structure of the family $\mathfrak{H}(R_n, S_m)$ of projections of analytic mappings of R_n into S_m.

Received September 11, 1984