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THE CONNECTION BETWEEN THE SYMMETRIC SPACE $E_6/SO(10)\cdot SO(2)$ AND PROJECTIVE PLANES

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Introduction.

We are interested in symmetric spaces of the three types EIII, EVI and EVIII on the same line as projective planes, since we want to understand all exceptional Lie groups geometrically and systematically. The first aim in this paper is to clarify the structure of maximal flat tori in a compact symmetric space Π of the type EIII which is constructed in the set of projections associated with involutive automorphisms of the compact simple Lie algebra of the type E_6 (Theorem 3.6). Next we write down the roots of the symmetric space Π and also give a relation between the roots and the isotropy groups of two points in Π. In Section 5 two objects, points and lines, are introduced into Π, and this space is studied from the viewpoint of projective geometry. Finally it is showed that Π is a projective plane in the wider sense (Theorem 5.10).

1. Preliminaries.

Let $\mathfrak{A}$ be a composition algebra over the real field $\boldsymbol{R}$ and let a, b, c be elements in $\mathfrak{A}$. If a conjugation $-: a\to\bar{a}$ is usually defined in $\mathfrak{A}$, we have a symmetric inner product $(a, b)=1/2(a b+\overline{a b})$. If a commutator and an associator are defined by $[a, b]=ab-ba$ and $(a, b, c)=(ab)c-a(bc)$ respectively, any inner derivation of $\mathfrak{A}$ can be generated by $D_{a,b}$, where $D_{a,b}(c)=[[a, b], c]-3(a, b, c)$.

Let $\mathfrak{A}^{(1)}\otimes M^3\otimes\mathfrak{A}^{(2)}$ denote an tensor product over $\boldsymbol{R}$ composed of one 3×3 matrix algebra M^3 with coefficients in $\boldsymbol{R}$ and two composition algebras $\mathfrak{A}^{(i)}$. If the confusion does not occur, we write aXu instead of $a\otimes X\otimes u$, where $a\in\mathfrak{A}^{(1)}$, $u\in\mathfrak{A}^{(2)}$ and $X\in M^3$. In this vector space a product can be defined by $xy=abXYuv$ for $x=aXu$ and $y=bYv$. Furthermore an involution and a trace Tr can be introduced by $aXu\to\bar{a}X^T\bar{u}$ and $Tr(aXu)=a\,\mathrm{tr}(X)Iu$ respectively, where $T: X\to X^T$ is the transposed operator of matrix, $\mathrm{tr}(X)=1/3(x_{11}+x_{22}+x_{33})$ for $X=(x_{ij})\in M^3$, and I is the 3×3 unit matrix.

Let $\mathfrak{M}$ denote a vector space over $\boldsymbol{R}$ which is generated by all elements in $\mathfrak{A}^{(1)}\otimes M^3\otimes\mathfrak{A}^{(2)}$ with the trace Tr being 0 and the skew-symmetric form with respect to the involution $aXu\to\bar{a}X^T\bar{u}$. Let $L(\mathfrak{A}^{(1)}, M^3, \mathfrak{A}^{(2)})$ be the vector space

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