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ON THE ELASTIC CLOSED PLANE CURVES

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§1. Introduction.

With respect to the total curvature of a closed curve C of class C^2 in a 3-dimensional Euclidean space E^3, we have the classical Fenchel inequality ([3] in 1929)

$$\int_C k(s)ds \geqq 2\pi\,, \tag{1.1}$$

where s denotes the arc length parameter of C and $k(s)$ the curvature of C. If a closed curve C is knotted in E^3, then the Fary inequality

$$\int_C k(s)ds \geqq 4\pi \tag{1.2}$$

holds good (cf. Fary [2] and J. Milnor [5]).

If a closed curve C is regarded as an elastic rod, then the bending energy $E(C)$ of the deflected curve C from $k=0$ is given by (cf. [4], [8])

$$E(C)=\frac{1}{2}\int_C k^2(s)ds\,. \tag{1.3}$$

For any real number t, we get

$$0 \leqq \int_C (k(s)-t)^2 ds = \int_C k^2(s)ds - 2t\int_C k(s)ds + t^2\int_C ds\,.$$

Then, from (1.1) we obtain

$$E(C)=\frac{1}{2}\int_C k^2(s)ds \geqq 2\pi^2/L\,, \tag{1.4}$$

where L is the length of the closed curve C. The equality holds good if and only if C is a circle of radius $L/2\pi$ in the plane.

Concerning the inequality (1.4), I. Bives ([1], p. 283) showed the following:

Let M be a circle of radius r, isometrically immersed into E^N. If k denotes the curvature function, then

$$\int_N k^2(s)ds \geqq 2\pi/r \tag{1.5}$$

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