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POINTWISE CONVERGENCE OF THE PRODUCT INTEGRAL FOR A CERTAIN INTEGRAL TRANSFORMATION ASSOCIATED WITH A RIEMANNIAN METRIC

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§ 0. Introduction.

In [13] Inoue-Maeda present a rigorous meaning to the convergence of the path integral in a non-compact curved space. Though comparing with Feynman's original idea, they considered the case where $i\hbar^{-1}$ is replaced by $-\lambda$ $(\lambda>0)$. Namely, they considered a certain integral transformation associated with a given Lagrangian function of the form; $L(x, \dot{x})=g_{ij}(x)\dot{x}_i\dot{x}_j+V(x)$, where $G=(g_{ij}(x))$ defines a Riemannian metric and $V(x)$ is a smooth function with the compact support, and showed the convergence of its product integral in the *topology of the uniform operator norm.*

The purpose of this paper is to continue the above work as follows; First, we extend the above integral transformations to those which acts on sections of a general vector bundle (Cf. (0.1)). Also, we construct fundamental solutions for parabolic systems geometrically. Here, we shall deal with the case $V=0$, only for simplicity. The second aim is to show the convergence of the product integral of the integral transformation in a refined topology (*pointwise convergence of the kernel function*).

We suspect that these observation for the convergence of the product integral may have interesting applications, and here we can derive the asymptotic behavior of the fundamental solution for a parabolic system defined on the non-compact manifold in terms of geometrical invariants.

Let (M, g) be a smooth, complete m-dimensional Riemannian manifold and let E be a vector bundle over M with a linear connection D. Suppose that E is furnished with an inner product $\langle\,,\rangle_x$ at each fibre E_x, $x\in M$, preserved by D. Using the connection D, we can consider the parallel translation along the minimal geodesic γ_c from y to x, which maps an element of E_y to that of E_x. We denote it by $P(x, y)$ (Cf. § 2).

Denote by $C_0(E)$ the set of all continuous sections of E with compact support and by $C^\infty(E)$ that of all smooth sections of E. Put $C_0^\infty(E)=C_0(E)\cap C^\infty(E)$.

For $\xi\in C_0^\infty(E)$, we define the L^2-norm as

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