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HELICAL IMMERSIONS AND NORMAL SECTIONS

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1. Introduction.

Let $f: M^n \to \bar{M}^{n+p}$ be an isometric immersion of a connected n-dimensional Riemannian manifold M into a Riemannian manifold $\bar{M}$ of dimension $n+p$. If $\gamma: I=[0, 1] \to M$ is a curve on M then $\sigma = f \circ \gamma$ is a curve on $\bar{M}$. Let σ be parametrized by its arc length, $\sigma^{(1)} = \dot{\sigma}$ be the unit tangent vector and $K_1 = \|\tilde{\nabla}_{\dot{\sigma}} \sigma^{(1)}\|$. $\tilde{\nabla}$ denotes the covariant differentiation of $\bar{M}$. If K_1 vanishes on $[0, 1]$ then σ is called of order 1. If K_1 is not identically zero, then we define $\sigma^{(2)}$ by $\tilde{\nabla}_{\dot{\sigma}} \sigma^{(1)} = K_1 \sigma^{(2)}$ on the set $I_1 = \{s \in [0, 1] : K_1(s) \neq 0\}$. Let $K_2 = \|\tilde{\nabla}_{\dot{\sigma}} \sigma^{(2)} + K_1 \sigma^{(1)}\|$. If $K_2 \equiv 0$ on I_1 then σ is called of order 2. If K_2 is not identically zero on I_1 then we define $\sigma^{(3)}$ by $\tilde{\nabla}_{\dot{\sigma}} \sigma^{(2)} = -K_1 \sigma^{(1)} + K_2 \sigma^{(3)}$. Inductively we put $K_d = \|\tilde{\nabla}_{\dot{\sigma}} \sigma^{(d)} + K_{d-1} \sigma^{(d-1)}\|$. If $K_d \equiv 0$ on I_{d-1} then σ is called of order d. It follows that if the curve σ is of order d we have the Frenet formula ([9]):

$$\tilde{\nabla}_{\dot{\sigma}}(\sigma^{(1)}, \sigma^{(2)}, \cdots, \sigma^{(d)}) = (\sigma^{(1)}, \sigma^{(2)}, \cdots, \sigma^{(d)})K \tag{1.1}$$

where

$$K = \begin{bmatrix} 0 & -K_1 & 0 & \cdots & 0 \\ K_1 & 0 & -K_2 & & 0 \\ 0 & K_2 & 0 & \ddots & \\ & 0 & \ddots & & -K_{d-1} \\ & & & K_{d-1} & 0 \end{bmatrix}$$

$K_1, K_2, \cdots, K_{d-1}$ are called the Frenet curvatures of σ. If, for each geodesic γ on M, the curve $f \circ \gamma$ on $\bar{M}$ has constant Frenet curvatures of order d, and they are independent of γ, then f is called a helical immersion of order d. In most cases the ambient space is considered as a Riemannian manifold of constant sectional curvature c, denoted by $\bar{M}^{n+p}(c)$. Sakamoto [9] and Nakagawa [8] have investigated helical immersions. The concept "helical immersion" originates from Besse [2]; it is important in the theory of harmonic manifolds.

Another important concept used in this paper called normal sections, originated from Chen [3]. In [3], [4], [7], submanifolds in E^m with (pointwise) planar normal sections were investigated. Chen and Verheyen [5] proved that

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