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IMAGE AREAS AND BMO NORMS OF ANALYTIC FUNCTIONS

Dedicated to Professor Yukio Kusunoki on his 60th birthday

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1. Introduction. Let R be a nonparabolic open Riemann surface and $g(z, a)$ denote its Green's function with logarithmic singularity at $a\in R$. For a function f analytic on R, we define

$$A(f)=\frac{1}{\pi}\text{area}\{f(R)\},$$

$$B(f)=\sup_{a\in R}\frac{2}{\pi}\iint_R|f'(z)|^2 g(z, a)dxdy \tag{1.1}$$

and

$$D(f)=\frac{1}{\pi}\iint_R|f'(z)|^2 dxdy,$$

where $z=x+iy$ denotes a local coordinate on R. We consider following spaces of analytic functions on R:

$$BMOA(R)=\{f: B(f)<+\infty\}, \tag{1.2}$$

$$AD(R)=\{f: D(f)<+\infty\}.$$

Metzger [10] introduced $BMOA(R)$ by (1.1) and (1.2) and showed the inclusion relation $AD(R)\subset BMOA(R)$ by using a celebrated result of Hayman and Pommerenke [3]. Stegenga [13] independently obtained a similar result as theirs and remarked as an easy consequence that the inequality

$$B(f)\leqq cA(f) \tag{1.3}$$

with some constant c holds for functions f analytic in the unit disc U of the complex plane $\boldsymbol{C}$. Recently the author [8] showed that (1.3) holds with $c=1$, that is, the inequality

$$B(f)\leqq A(f) \tag{1.4}$$

holds for functions f analytic on R, which obviously implies

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