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NON-INTEGRABILITY OF HÉNON-HEILES SYSTEM AND A THEOREM OF ZIGLIN

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1. Introduction.

This paper concerns the integrability of Hamiltonian systems with two degrees of freedom

$$\dot{q}_k = H_{p_k}, \quad \dot{p}_k = -H_{q_k} \qquad (k=1, 2), \tag{1.1}$$

where the dot indicates the differentiation with respect to time variable t. We assume that the Hamiltonian H is of the form

$$H = H(q, p) = \frac{1}{2}|p|^2 + V(q); \quad |p|^2 = p_1^2 + p_2^2, \tag{1.2}$$

where $V(q)$ is a polynomial of q_1 and q_2. We consider this system in the complex domain. A single-valued function $F(q, p)$ is called an *integral* of (1.1) if it is constant along any solution curve $(q(t), p(t))$ of (1.1). This implies that $(d/dt)F(q(t), p(t))=0$, which leads to the identity

$$\sum_{k=1}^{2}(F_{q_k}H_{p_k} - F_{p_k}H_{q_k}) = 0. \tag{1.3}$$

In particular, the Hamiltonian H is an integral. In this paper, the system (1.1) is said to be *integrable* if there exists an *entire* integral F which is functionally independent of H.

From the viewpoint of dynamical systems, our interest is in the behavior of real solutions for real analytic Hamiltonian systems. However, in the majority of integrable problems of Hamiltonian mechanics, the known integrals can be extended to the complex domain. Therefore, it is natural to discuss the integrability of complex Hamiltonian systems in the above sense, that is, the existence of additional entire integrals other than the Hamiltonian. Moreover, a new aspect appears from considering solutions in complex time plane. It is the branching of solutions as functions of time variable t. In general, the solutions branch in finite or infinite manner by analytic continuation. In this paper, we discuss the integrability of (1.1) in connection with the branching of solutions.

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