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ON CONVERGENCE OF CONDITIONAL PROBABILITY MEASURES

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1. Introduction.

In this paper we are concerned with a relation between convergence of conditional probability measures given sample means and minimization of I-divergence (Kullback-Leibler information quantity) under some constraint. For statistical and information-theoretical meanings of this problem, we refer to Vincze [15]. Similar problem was investigated by Bártfai [3] and Vasicek [14], but our approach is slightly different from theirs. We apply Sanov-type theorems which were obtained by Groeneboom, Oosterhoff and Ruymgaart [6]. They were working in the intrest of rates of convergence for probabilities of large deviations given sample means. We recognize their result as a limit of average information quantity gained by measurement of sample means.

The basic definitions and results which will be used in the sequal are provided in Section 2. The result of Section 3 is not so difficult, but it would be helpful for understanding the following work. Section 4 is our main one. At first we rewrite a large deviation theorem obtained in Groeneboom, Oosterhoff and Ruymgaart [6] employing I-divergence of conditional probability measures given sample means. From this point of view we can show convergence of conditional probability measures in the total variation metric. We also consider a problem of convergence in τ-topology.

2. Preliminaries.

The purpose of this section is to state the basic definitions and the principal results which will be used in what follows.

Let X be a Hausdorff space of points x, $\mathcal{B}$ the σ-field of Borel subsets of X. Let Π be the set of all probability measures on $(X, \mathcal{B})$, which is considered as a convex set in the usual sense: $(a\lambda+(1-a)\mu)(\cdot)=a\lambda(\cdot)+(1-a)\mu(\cdot)$, $0\leqq a\leqq 1$, λ, $\mu\in\Pi$. The *I-divergence or Kullback-Leibler information quantity* $I(\lambda|\mu)$ for λ, μ in Π is defined by

$$I(\lambda|\mu)=\int\log\frac{d\lambda}{d\mu}d\lambda \qquad \text{if} \quad \lambda\ll\mu\,,$$

$$=+\infty \qquad\qquad \text{otherwise.}$$

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