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GROWTH OF COMPOSITE ENTIRE FUNCTIONS

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Introduction. If f and g are transcendental entire functions then Clunie [1] proved that $\lim_{r\to\infty}\frac{T(r, f(g))}{T(r, f)}=\infty$. An obvious question arises is, what can be said about the ratio

$$\frac{\log T(r, f(g))}{T(r, f)} \tag{1}$$

when $r\to\infty$? In general by considering $g(z)=e^{e^z}$, and $f(z)=e^z$, we see that the ratio (1) also tends to infinity. However if we put some restriction on the orders of f and g then we can show that the above ratio is bounded above by a finite quantity. Thus the purpose of this paper will be to prove some results dealing with the ratios that are of the form (1). We start with

THEOREM 1. *Let $f(z)$ and $g(z)$ be entire functions of finite order such that $g(0)=0$ and $\rho_g<\lambda_f\leqq\rho_f$ where ρ, λ denote respectively the order and the lower order for the corresponding functions. Then*

$$\limsup_{r\to\infty}\frac{\log T(r, f(g))}{T(r, f)}\leqq\rho_f .$$

Note. (i) From the hypothesis it is clear that f must necessarily be transcendental.

(ii) The theorem does not hold true when $\rho_g=\rho_f$, for let $f(z)=e^z$ and $g(z)=e^z-1$, then $\rho_g=\rho_f=1$ and $T(r, f(g))\sim\frac{e^r}{(2\pi^3 r)^{1/2}}$ see [2, 7], so that

$$\limsup_{r\to\infty}\frac{\log T(r, f(g))}{T(r, f)}=\pi .$$

(iii) In case $\rho_g>\rho_f$ we shall show that the limit superior will tend to infinity. Thus we shall prove

THEOREM 2. *Let $f(z)$ and $g(z)$ be entire functions of finite order with $\rho_g>\rho_f$. Then*

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