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AN INTRINSIC FIBRE METRIC ON THE n-TH SYMMETRIC TENSOR POWER OF THE TANGENT BUNDLE

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0. Introduction. Let $H(M)$ be the Hilbert space consisting of all square-integrable holomorphic m-forms on an m-dimensional complex manifold M. The Bergman form K is defined as a specific holomorphic $2m$-form on the product manifold $M\times\overline{M}$, where $\overline{M}$ is the conjugate complex manifold of M. Let $z=(z^1, \cdots, z^m)$ be a coordinate system with defining domain U_z, and k_z be the Bergman function relative to z, i.e. $K(p, \bar{p})=k_z(p)(dz^1\wedge\cdots\wedge dz^m)_p\wedge(d\bar{z}^1\wedge\cdots\wedge d\bar{z}^m)_{\bar{p}}$, $p\in U_z$. In general, $k_z\geqq 0$. In Kobayashi [4], the following conditions are considered:

(A.1) For every $p\in M$, there exists $\alpha\in H(M)$ such that $\alpha(p)\neq 0$.

(A.2) For every non-zero tangent vector X at $p\in M$, there exists $\alpha\in H(M)$ such that $\alpha(p)=0$ and $X.\alpha(p)\neq 0$.

Suppose (A.1) holds. Then $k_z>0$ for every z, and the Bergman pseudo-metric g, with components $g_{a\bar{b}}=\partial_a\overline{\partial_b}.\log k_z$, is defined. Furthermore, the following is known ([4]):

(K_1) g is a metric if and only if (A.2) holds.

If M satisfies (A.1) and (A.2), and if $R_{a\bar{b}c\bar{d}}$ are the components of the hermitian curvature tensor of the Bergman metric, then the following are known ([4]):

(K_2) Set $\hat{R}_{ac\overline{bd}}=R_{a\bar{b}c\bar{d}}+g_{a\bar{b}}g_{c\bar{d}}+g_{a\bar{d}}g_{c\bar{b}}$. Then $\sum \hat{R}_{ac\overline{bd}}v^a v^c \bar{v}^b \bar{v}^d\geqq 0$ for every $(v^1, \cdots, v^m)\in \boldsymbol{C}^m$.

(K_3) $\hat{R}_{ac\overline{bd}}=k^{-1}(k_{ac\overline{bd}}-k^{-1}k_{ac}k_{\overline{bd}})-k^{-2}\sum g^{\bar{t}s}(k_{ac\bar{t}}-k^{-1}k_{ac}k_{\bar{t}})(k_{s\overline{bd}}-k^{-1}k_{\overline{bd}}k_s)$, where $k=k_z$, $k_{ac}=\partial_a\partial_c.k$, etc., and $(g^{\bar{t}s})=(g_{a\bar{b}})^{-1}$.

In the preceding joint paper [2] with Burbea, conditions (C_n) are defined so that (C_0) (resp. (C_1)) coincides with (A.1) (resp. (A.2)). Furthermore, under assumption (C_0), non-negative functions $\mu_{0,n}$, which are biholomorphic invariants, on the tangent bundle are introduced.

In the present paper, we first note (Proposition 1.2) that the functions $\mu_{0,n}$ on the tangent bundle are, in general, upper semi-continuous, and show (Theorem 2.1) that when M satisfies condition (C_0) there exists a unique fibre pseudo-metric $g^{(n)}$ on the n-th symmetric tensor power $S^nT(M)$ of the tangent bundle

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