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ON AN ESTIMATE FOR $\int_0^\infty m(t, E(-z, q))t^{-1-\beta}dt$

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In this paper we shall give a lower estimate for

$$\int_0^\infty \frac{m(t,\ E(-z,\ q))}{t^{1+\beta}}\,dt,$$

where $E(z, q)$ is the Weierstrass primary factor of genus q, β a constant satisfying $q<\beta<q+1$ and $m(t, f)$ the Nevanlinna proximity function. Our result is the following

THEOREM.

$$\int_0^\infty \frac{m(t,\ E(-z,\ q))}{t^{1+\beta}}\,dt > \frac{1}{\beta^2\kappa(\beta)},$$

where $\kappa(\beta)$ *is the constant defined by*

$$\begin{cases} \dfrac{|\sin \pi\beta|}{q+|\sin \pi\beta|} & (q<\beta<q+1/2), \\ \dfrac{|\sin \pi\beta|}{q+1} & (q+1/2\leqq\beta<q+1). \end{cases}$$

In the above estimation equality does not occur. In order to show this inequality part we need a rough tracing of the level curve

$$\log|1+te^{i\theta}|+\sum_{j=1}^{q}(-1)^j j^{-1}t^j \cos j\theta=0.$$

However we do not need its precise analysis.

Proof. Let us consider

$$I_F=\int_0^\infty \frac{1}{\pi}\int_F \log|E(-te^{i\theta},\ q)|\,d\theta\,\frac{dt}{t^{1+\beta}},$$

where F is a measurable subset of $[0, \pi]$. Evidently

$$I_F\leqq\int_0^\infty \frac{m(t,\ E(-z,\ q))}{t^{1+\beta}}\,dt$$

for any F. Further it is known [1] that it is possible to change the order of

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