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A METHOD TO A PROBLEM OF R. NEVANLINNA, II

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§1. **Introduction.** This paper is a continuation of our earlier paper [3]. In this paper we shall prove the following theorems.

THEOREM 1. *Let $f(z)$ be a meromorphic function of regular growth of order ρ. Then*

$$K(f)\geqq L(\rho)\liminf_{t\to\infty} S(t, E)/T(t, f).$$

THEOREM 2. *Let $f(z)$ be a meromorphic function defined by a quotient of two canonical products of genus q*

$$f(z)=\prod E\left(\frac{z}{a_n}, q\right)\Big/\prod E\left(\frac{z}{b_n}, q\right).$$

Suppose that the order λ and the lower order μ of $f(z)$ satisfies $q\leqq\mu<\lambda<q+1$. Let β be a number satisfying $\mu<\beta<\lambda$. Them for any E

$$\sup_{\mu<\beta<\lambda} L(\beta)\liminf_{t\to\infty} S(t, E)/T(t, f)\leqq K(f).$$

Theorem 2 was already stated without proof in [3].

In order to prove Theorem 1 we make use of the notion of proximate order of $T(t, f)$. The proximate order $l(t)$ is defined by the following conditions:

(i) $l(t)$ is real continuous and piecewise differentiable for $t>t_0$,

(ii) $l(t)\to\rho$ as $t\to\infty$,

(iii) $tl'(t)\log t\to 0$ as $t\to\infty$,

(iv) $\limsup_{t\to\infty}\frac{T(t, f)}{t^{l(t)}}=1$.

Let us put

$$\mu(t)=t^{\rho-l(t)},$$

then $\mu(t)$ is a slowly varying function in the sense of Karamata, that is, $\mu(t)$ satisfies $\mu(ct)/\mu(t)\to 1$ as $t\to\infty$ for any positive c. It is known that the above convergence is uniform in the wider sense in $(0, \infty)$. See Seneta [5]. Then it is easy to prove that

$$\int_{t_0}^{\infty} T(t, f)t^{-1-l(t)}dt=\infty$$

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