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A METHOD TO A PROBLEM OF R. NEVANLINNA

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§1. **Introduction.** This paper is concerned with a problem posed by R. Nevanlinna in his monumental paper [6] and successively in his treatise on meromorphic functions [7]. He proved the following theorem.

THEOREM A. *Let $f(z)$ be a meromorphic function in $|z|<\infty$ and let*

$$K(f)=\limsup_{t\to\infty}\frac{N(t,0)+N(t,\infty)}{T(t,f)}.$$

Then there is a constant $C(\rho)$ such that for a non-integral order ρ of f

$$K(f)\geqq C(\rho)>0.$$

Simultaneously he made the following conjecture:

$$\kappa(\rho)\equiv\inf K(f)=\begin{cases}\dfrac{|\sin\pi\rho|}{q+|\sin\pi\rho|} & (q\leqq\rho<q+1/2),\\[2mm] \dfrac{|\sin\pi\rho|}{q+1} & (q+1/2\leqq\rho\leqq q+1),\end{cases}$$

where inf is taken over all meromorphic functions f of order ρ.

Edrei and Fuchs [2] proved

$$\kappa(\rho)=\begin{cases}1 & (0\leqq\rho<1/2),\\ \sin\pi\rho & (1/2\leqq\rho\leqq 1).\end{cases}$$

Goldberg's lemma played the decisive role in their paper. Hellerstein and Williamson [5] proved that the conjecture is true for entire functions of order ρ with only negative zeros. They made use of Shea's representation and of a very precise analysis of the given function.

Through this paper we shall restrict to the following meromorphic function $f(z)$ defined by a quotient of two canonical products

$$f(z)=f_1(z)/f_2(z),$$

$$f_1(z)=\prod E(z/a_n, q),\qquad f_2(z)=\prod E(z/b_n, q).$$

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