S. SATO
KODAI MATH. J.
8 (1985), 5—13

AN INEQUALITY FOR THE SPECTRAL RADIUS OF MARKOV PROCESSES

BY SADAO SATO

1. Introduction.

Let A be a second-order uniformly elliptic operator in a bounded domain D. Consider the eigenvalue problem

$$Au+\lambda u=0 \tag{1.1}$$

with mixed boundary conditions:

$$\begin{aligned} u&=0 \quad \text{on } \Gamma_1 \\ \frac{\partial u}{\partial n}+\alpha(x)u&=0 \quad \text{on } \Gamma_2, \end{aligned} \tag{1.2}$$

where n stands for the outer normal and $\partial D=\Gamma_1 \cup \Gamma_2$. Let λ_0 be the first eigenvalue. When A is symmetric, J. Barta proved that

$$\inf\{-Au/u\} \leqq \lambda_0 \leqq \sup\{-Au/u\}, \tag{1.3}$$

where u is any positive C^2-function satisfying the same boundary conditions (1.2) (see [1]).

When A is nonsymmetric, M. H. Protter and H. F. Weinberger [7] proved the left hand of (1.3) for any function u satisfying

$$\begin{aligned} u&>0 \quad \text{on } D\cup\partial D \\ \frac{\partial u}{\partial n}+\alpha(x)u&\geqq 0 \quad \text{on } \Gamma_2. \end{aligned} \tag{1.4}$$

Let $\alpha(x)$ be positive. Then there exists a diffusion process with the generator A whose domain is the collection of C^2-functions satisfying (1.2).

For a Markov process, we can define the spectral radius λ_0 by

$$\lambda_0=\lim_{t\to\infty}-\frac{1}{t}\log\|T_t\|, \tag{1.5}$$

where $\{T_t\}$ is the associated semigroup and $\|T_t\|=\sup_x T_t 1(x)$.

Our main purpose is to prove the inequality (1.3) for the spectral radius of a Markov process satisfying some conditions. We will show that the spectral

Received January 31, 1984