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ON THE SUMMABILITY ALMOST EVERYWHERE OF THE MULTIPLE FOURIER SERIES AT THE CRITICAL INDEX

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Recently Stein [5] shows the existence of an H^1 function on the k-torus, whose Fourier series is almost everywhere non-summable with respect to the Bochner-Riesz means of the critical index $(k-1)/2$.

On the background of this example, he says as follows. In the case $k=1$, It is known that there exists an H^1 function whose Fourier series diverges almost everywhere (see [6], [7]). In the other direction the theorem of Carleson-Hunt-Sjölin guarantees the convergence almost everywhere whenever $f\in L\log L(\log\log L)$; see [3].

For the multiple Fourier series, S. Bochner pointed out that summability at the critical index $(k-1)/2$ is the correct analogue of convergence for phenomena near L. In this sense Stein's example is a version in the case of general k. On the other hand, as another version he says that whenever $f\in L(\log L)^2$, the multiple Fourier series of f is summable almost everywhere at the critical index $(k-1)/2$. However this version is slightly different from the one dimensional case. The purpose of this note is to show that we can replace the last condition by $f\in L(\log L)(\log\log L)$.

Let $f(x)=f(x_1, x_2, \cdots, x_k)\in L$ on $Q_k: -\pi<x_i\leqq\pi$ $(i=1, 2, \cdots, k)$ and its Fourier series be

$$f(x)\sim\sum a_n e^{in\cdot x}$$

where

$$a_n=(2\pi)^{-k}\int_{Q_k} f(x)e^{-in\cdot x}dx\,.$$

Then the δ-th Bochner-Riesz means of the series is

$$(S_R^\delta f)(x)=\sum_{|n|<R}(1-|n|^2/R^2)^\delta a_n e^{in\cdot x}.$$

THEOREM. *If* $\int_{Q_k}|f(x)|(\log^+|f(x)|)(\log^+\log^+|f(x)|)dx<\infty$, *then,*

$$\lim_{R\to\infty}(S_R^\alpha f)(x)=f(x) \qquad \text{a.e.},$$

where $\alpha=(k-1)/2$ $(k>1)$.

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