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THE FUNDAMENTAL SOLUTIONS OF THE HEAT EQUATIONS ON RIEMANNIAN SPACES WITH CONE-LIKE SINGULAR POINTS

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§ 0. Introduction.

The purpose of this paper is to derive some properties of the fundamental solution of the initial-value problem

$$
(0.1) \qquad \begin{aligned} &\left(\frac{\partial}{\partial t}+\Delta\right)\theta(t, x)=0, \qquad t>0 \\ &\lim_{t\downarrow 0} \theta(t, x)=\theta(x) \end{aligned}
$$

for forms on a Riemannian space with cone-like singular points. Here the Laplacian Δ is of Neumann or Dirichlet types, or, in certain cases, of unphysical types with ideal boundary conditions. We can show that the asymptotic expansion of the trace of the fundamental solution can have the log term and therefore the zeta function can have the simple pole at the origin; these new phenomena arising from the existence of the singular points will evoke much interest. In order to investigate them more closely, we will further study the same problem on the metric cone with the help of the Fourier integral operator theory.

The direction of this investigation has been first raised up by a short but pioneering paper due to J. Cheeger ([2]). He attempted to extend the spectral geometric theory to the case where manifolds have singularities. The author's study substantially follows in his direction and should be started with carrying out the above basic research.

Notations and definitions: Before explaining the contents of this paper in detail, we will collect the general notations and definitions.

First, let Y be a (perhaps, incomplete) oriented Riemannian manifold with $\widehat{\partial Y}\subset Y$; $Y=\operatorname{Int} Y\cup\widehat{\partial Y}$ and, for the metric completion $\overline{Y}$,

$$
(0.2) \qquad \widehat{\partial Y}=\left\{y\in \overline{Y}-\operatorname{Int} Y \,\middle|\, \begin{array}{l}\overline{Y} \text{ is a manifold with smooth boundary} \\ \text{in some neighborhood of } y.\end{array}\right\}.
$$

Let $\Lambda^i=\Lambda^i(\operatorname{Int} Y)$ denote the space of smooth i-forms on Int Y and let $\Lambda_0^i=\Lambda_0^i(\operatorname{Int} Y)$ be the subspace of Λ^i consisting of forms of compact support. Let $\Lambda^i(Y)$ be the subspace of Λ^i consisting of forms which are smooth up to $\widehat{\partial Y}$. For

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