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KODAI MATH. J.
7 (1984), 365—381

SELF-HOMOTOPY EQUIVALENCES OF THE TOTAL SPACES OF A SPHERE BUNDLE OVER A SPHERE

Dedicated to Professor Minoru Nakaoka on his 60th birthday

BY SEIYA SASAO

§ 1. Introduction.

In this paper we study the group of homotopy classes of self-homotopy equivalences, $\mathcal{E}(X)$, for the total space of a S^m-bundle over S^n with the condition:

$$3<m+1<n<2m-2.$$

J. W. Rutter determined this group for the case of $m=3$ and $n=7$ in [3], and also some generalizations of Rutter's result are given in [4] and [6]. Moreover Y. Nomura computed $\mathcal{E}(X)$ for real and complex Stiefel manifolds in [5]. Then our purpose is to obtain a generalization of these results in a some sense. Let H be the natural representation:

$$H:\mathcal{E}(X)\longrightarrow \mathrm{Aut}\, H_*(X)$$

which is defined by $H(f)=f_*$ and we denote by $\mathcal{E}_+(X)$ the kernel of H. Then we have an exact sequence

$$\{1\}\longrightarrow \mathcal{E}_+(X)\longrightarrow \mathcal{E}(X)\underset{H}{\longrightarrow} \mathrm{Aut}\, H_*(X).$$

Hence it is almost sufficient for us to determine $\mathcal{E}_+(X)$ and H-image.

Let $q: X\to S^n$ be the S^m-bundle with the characteristic class $\xi\ (\in\pi_{n-1}(SO(m+1)))$. James-Whitehead showed in [2] that X has a CW-decomposition:

$$X=S^m\underset{\beta}{\bigcup} e^n \underset{\alpha}{\bigcup} e^{m+n},$$

where $\beta=p_*(\xi)$ for the usual projection $p: SO(m+1)\to S^m$.

Let $P_n^m(\beta)$ be the subgroup of $\pi_n(S^m)$,

$$\{x\,|\,[\iota_n, x]\in\beta\circ\pi_{m+n-1}(S^{n-1})\},$$

and we denote by η the generator of $\pi_{N+1}(S^N)$. We will prove

Received January 30, 1984