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A CERTAIN PROPERTY OF GEODESICS OF THE FAMILY OF RIEMANNIAN MANIFOLDS O_n^2 (VI)

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§ 0. Introduction.

This is exactly a continuation of Part (V) ([15]) with the same title written by the present author which proved the following conjecture is true for $16 \leqq n \leqq 84$. On the methods used in it, the lower bound 16 of this effective interval is crucial from the argument in it. We shall show that this conjecture is also true for $9.7 \leqq n \leqq 16$ in the present paper by improving them. We shall use the same notation in the previous ones Parts (I)$\sim$(V).

The period T of any non-trivial solution $x(t)$ of the non-linear differential equation of order 2:

$$\text{(E)} \qquad n x(1-x^2)\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2+(1-x^2)(nx^2-1)=0$$

with a constant $n>1$ such that $x^2+x'^2<1$ is given by the integral:

$$\text{(0.1)} \qquad T=\sqrt{nc}\int_{x_0}^{x_1}\frac{dx}{x\sqrt{(n-x)\{x(n-x)^{n-1}-c\}}},$$

where $x_0=n\{\min x(t)\}^2$, $x_1=n\{\max x(t)\}^2$, $0<x_0<1<x_1<n$ and $c=x_0(n-x_0)^{n-1}=x_1(n-x_1)^{n-1}$.

CONJECTURE C. The period T as a function of $\tau=(x_1-1)/(n-1)$ and n is monotone decreasing with respect to $n(>2)$ for any fixed $\tau(0<\tau<1)$.

Here the author thanks heartily Professor Naoto Abe for his cooperation in the numerical computations by computors.

§ 1. The fundamental principle to attain the purpose.

Setting $T=\Omega(\tau, n)$, we have the formulas

$$\text{(1.1)} \qquad \frac{\partial\Omega(\tau, n)}{\partial n}=-\frac{\sqrt{c}}{2b^2\sqrt{n}}\int_{x_0}^{1}\frac{(1-x)W(x, x_1)dx}{x(n-x)\sqrt{x(n-x)^{n-1}-c}}$$

(Lemma 3.1 in (III)) and

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