S. KIMURA
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A CHARACTERIZATION OF THE EXPONENTIAL FUNCTION BY PRODUCT II

BY SHIGERU KIMURA

§1. **Introduction.** In our previous paper [2] we proved the following result.

THEOREM A. *Suppose that $f(z)$ is an entire function of order $q=2p+1$ having only negative zeros. Setting $\phi(z^2)=f(z)f(-z)$, $g(z)=\phi(-z)/\phi(0)$, we assume that $g(z)$ is a canonical product. Further we assume that there is an arbitrarily small $\beta>0$ such that if $|g(r)|\geqq 1$,*

$$\log|g(re^{i\beta})|\leqq(\cos\beta q/2)\log|g(r)|$$

for all sufficiently large r and if $|g(r)|\leqq 1$,

$$\log|g(re^{i\beta})|\geqq(\cos\beta q/2)\log|g(r)|$$

for all sufficiently large r. Then $f(z)=e^{P(z)}$ where $P(z)$ is a polynomial of degree q, or else

$$\lim_{r\to\infty}\frac{\log M(r, f)}{r^q}=+\infty .$$

The purpose of this paper is to improve Theorem A and prove the following.

THEOREM. *Suppose that $f(z)$ is an entire function of order $q=2p+1$ having only negative zeros. Setting $\phi(z^2)=f(z)f(-z)$, $g(z)=\phi(-z)/\phi(0)$, we assume that there is an arbitrarily small $\beta>0$ such that if $|g(r)|\geqq 1$ for all sufficiently large r,*

$$\log|g(re^{i\beta})g(re^{-i\beta})|\leqq 2(\cos\beta q/2)\log|g(r)| \tag{1}$$

for all sufficiently large r and if $|g(r)|\leqq 1$ for all sufficiently large r,

$$\log|g(re^{i\beta})g(re^{-i\beta})|\geqq 2(\cos\beta q/2)\log|g(r)| \tag{2}$$

for all sufficiently large r. Then $f(z)=e^{P(z)}$ where $P(z)$ is a polynomial of degree q, or else

$$\lim_{r\to\infty}\frac{\log M(r, f)}{r^q}=+\infty . \tag{3}$$

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