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ON REAL HYPERSURFACES OF FINITE TYPE OF CP^m

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§ 1. Introduction.

Let M be a closed Riemannian manifold and Δ the Laplace-Beltrami operator of M acting on the smooth functions $C^\infty(M)$. It is well known that Δ is an elliptic operator with a discrete sequence of eigenvalues $0=\lambda_0<\lambda_1<\lambda_2<\cdots<\lambda_k<\cdots\uparrow\infty$. Let V_k be the eigenspace corresponding to the eigenvalue λ_k. Then V_k has finite dimension. Moreover the decomposition is orthogonal respect to the inner product

$$(f, g)=\int_M fgdV \tag{1.1}$$

and $\sum_k V_k$ is dense in $C^\infty(M)$.

Let $x : M\to E^m$ be an isometric immersion of M into the m-dimensional Euclidean space with coordinate functions x_i, that is, $x=(x_1, \cdots, x_m)$. Then for any $i=1, \cdots, m$, we have the decomposition

$$x_i=\sum_k (x_i)_k \qquad (L^2\text{-sense}). \tag{1.2}$$

As M is closed, V_0 consists of the constant functions on M and so, from (1.2) we can write

$$x_i-(x_i)_0=\sum_{k=p_i}^{q_i}(x_i)_k \tag{1.3}$$

where $q_i=\{\mathrm{Sup}\, k\,|\,(x_i)_k\neq 0\}$ (respectively, $p_i=\{\mathrm{Inf}\, k\,|\,(x_i)_k\neq 0\}$).

If $p=\mathrm{Inf}_i\{p_i\}$ and $q=\mathrm{Sup}_i\{q_i\}$ using (1.3) we obtain the following spectral decomposition (in a vector form)

$$x-x_0=\sum_{k=p}^{q} x_k \tag{1.4}$$

where $x_k : M\to E^m$ are smooth for any k, q is an integer or $q=\infty$, x_0 is a constant and $\Delta x_k=\lambda_k x_k$. x_0 is called center of gravity of M.

We shall say that the immersion x is of *finite type* if $q<\infty$. If not it will be called of *no finite type* [5].

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