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ON AN ALGEBRAIZATION OF THE RIEMANN-HURWITZ RELATION

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Introduction.

In this paper we study the canonical representation $\mathrm{Aut}(M)\to GL(g, \boldsymbol{C})$ with the space of holomorphic differentials on M as its representation module, where M is a compact Riemann surface of genus $g\geqq 2$ (cf. (1.1)). For an automorphism group AG of M we denote its image by $R(M, AG)$. The $GL(g, \boldsymbol{C})$-conjugate class of $R(M, AG)$ appears as an invariant of the holomorphic family of Riemann surfaces which is defined by the subgroup of Teichmüller modular group corresponding to the pair (M, AG) (cf. [4], [5]). From such a point of view among others we consider it a problem to determine $R(M, AG)$'s.

In this paper we introduce two necessary conditions, which turn out (in § 2) sufficient in case $g=2$, for a finite subgroup G of $GL(g, \boldsymbol{C})$ to be conjugate to some $R(M, AG)$. In § 1 we make an algebraic formulation of the Riemann-Hurwitz relation, in terms of which one of our conditions is given. In fact we define the data of "ramification" for a (special type of) finite subgroup of $GL(g, \boldsymbol{C})$ and we show our formulation is valid in this case. In § 2 we introduce another condition on G that the character defined by G is of the form of the Eichler trace formula. It is known [6] that this condition is also sufficient in case where G is of prime order (and $g\geqq 2$). Using these two conditions, we determine 21 types of representatives (up to $GL(g, \boldsymbol{C})$-conjugacy) of $R(M, AG)$'s in the case $g=2$.

In a similar line we shall determine $R(M, AG)$'s in another place when $g=3$ (55 types) and $g=4$ (74 types).

Notation.

As usual $\boldsymbol{C}$ mean the field of complex numbers. The subgroup of a group generated by a family $\{A_1, \cdots, A_r\}$ of its elements is denoted by $\langle A_1, \cdots, A_r\rangle$. We write $\sharp X$ for the cardinality of a finite set X. And for an element A of a group we denote its order by $\sharp A$. If T is an element of $GL(g, \boldsymbol{C})$, T^* denotes the automorphism of $GL(g, \boldsymbol{C})$ sending A to $T^{-1}\cdot A\cdot T$ $(A\in GL(g, \boldsymbol{C}))$.

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