S. SASAO
KODAI MATH. J.
6 (1983), 167—173

$\mathcal{E}(X)$ FOR NON-SIMPLY CONNECTED H-SPACES

BY SEIYA SASAO

§ 0. Introduction.

Let X be a path-connected H-space with a unit x_0 and let $\mathcal{E}(X)$ be the group of homotopy classes of homotopy equivalences: $(X, x_0)\to(X, x_0)$. In the case of X being simply connected, D.M. Sunday, J.R proved that if $\mathrm{rank}(\pi_i(X))\geqq 2$, for some i, then $\varepsilon(X)$ contains a non abelian free subgroup (Theorem B-(2) of [3]). In this paper we investigate the case of an associative H-space X being not simply connected and having the homotopy type of a CW-complex.

THEOREM A. *There exists a splitting exact sequence:*

$$\{1\} \longrightarrow \nu^{-1}(1) \longrightarrow \varepsilon(X) \underset{\nu}{\longrightarrow} GL(n, Z) \longrightarrow \{1\},$$

where n is the rank of $\pi_1(X, x_0)$.

Especially, since $GL(n, Z)$ is not of finite rank for $n\geqq 2$ we have

COROLLARY. *If $\mathrm{rank}(\pi_1(X, x_0))\geqq 2$ then $\mathcal{E}(X)$ is not of finite rank.*

Next let $\mathcal{E}_H(X)$ be the subgroup of $\mathcal{E}(X)$ consisting of homotopy-homomorphisms (H-maps), then we have

THEOREM B. *If the natural homomorphism*

$$\pi_1(Z(X), x_0) \longrightarrow \pi_1(X, x_0)/Torsion$$

is onto, where $Z(X)$ denotes the homotopy-centre of X, then $\mathcal{E}_H(X)$ contains $GL(n, Z)$ as a semi-direct factor.

In addition, if we assume that $\pi_1(X, x_0)$ is torsion free we have

COROLLARY. *$\mathcal{E}(X)$ is isomorphic to the direct sum $GL(n, Z)\oplus K(X)$, where $K(X)$ denotes the kernel of the natural representation:*

$$\mathcal{E}_H(X) \longrightarrow \mathrm{Aut}(\pi_1(X, x_0))=GL(n, Z).$$

Received July 7, 1982