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ON SUBMANIFOLDS ALL OF WHOSE GEODESICS ARE CIRCLES IN A COMPLEX SPACE FORM

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0. Introduction.

First of all we recall the notion of circles in a Riemannian manifold $\tilde{M}$. A curve $x(t)$ of $\tilde{M}$ parametrized by arc length t is called a *circle*, if there exists a field of unit vectors Y_t along the curve which satisfies, together with the unit tangent vectors $X_t=\dot{x}(t)$, the differential equations: $\tilde{\nabla}_t X_t = kY_t$ and $\tilde{\nabla}_t Y_t = -kX_t$, where k is a positive constant and $\tilde{\nabla}_t$ denotes the covariant differentiation $\tilde{\nabla}$ with respect to X_t. Let p be an arbitary point of $\tilde{M}$. For a pair of orthonormal vectors X and $Y \in T_p\tilde{M}$ and for a given constant $k>0$, there exists a unique circle $x(t)$, defined for t near 0, such that

$$x(0)=p,\ X_0=X \quad \text{and} \quad (\tilde{\nabla}_t X_t)_{t=0}=kY\,.$$

If $\tilde{M}$ is complete, $x(t)$ can be defined for $-\infty<t<+\infty$ (for details, see [9]).

Recently, Sakamoto [12] studied submanifolds all of whose geodesics are circles in a real space form $\tilde{M}$. A Riemannian manifold of constant curvature is called a *real space form*. He showed the following.

THEOREM 0.1 [12]. *Let M be a submanifold in a real space form $\tilde{M}$. Then the following three conditions are equivalent.*

(I) *The submanifold M is nonzero isotropic and has parallel second fundamental form.*

(II) *Every geodesic in M is a circle in $\tilde{M}$.*

(III) *The submanifold M is planar geodesic and not totally geodesic.*

In this paper, here and in the sequel, the conditions (I), (II) and (III) stand for those of Theorem 0.1, unless otherwise stated.

Moreover, Sakamoto [12] has classified such submanifolds M in the Euclidean sphere S^m. Of course, the above three conditions are not equivalent in the case that the ambient manifold $\tilde{M}$ is a complex space form. A Kaehler manifold of constant holomorphic sectional curvature is called a *complex space form*.

When the ambient manifold $\tilde{M}$ is a complex projective space, Pak [11] classified submanifolds under the condition (III), and Nomizu [8] and Naitoh [6] under the condition (I). Due to their works, we see that the condition (III) is

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