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ANOTHER CONSTRUCTION OF REAL SIMPLE LIE ALGEBRAS

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Introduction.

In 1966, J. Tits [6] and E.B. Vinberg [7] made explicit models of exceptional simple Lie algebras independently. It is known that Tits' models contain all the real forms of these Lie algebras (cf. N. Jacobson [4]). In this paper we shall give first another construction of compact (real) simple Lie algebras which are isomorphic to Tits' and Vinberg's models in the case of exceptional Lie algebras. Furthermore we shall make all involutive automorphisms in each Lie algebra and also give all the real forms explicitly, corresponding to the involutive automorphisms, which are simpler than Tits' Lie algebras.

1. Preliminaries.

Let $\mathfrak{A}$ be a composition algebra over the field $\boldsymbol{R}$ of real numbers. Let a, b, c be elements in $\mathfrak{A}$. If a conjugation $-: a \rightarrow \bar{a}$ is usually defined in $\mathfrak{A}$, we have a symmetric inner product $(a, b)=1/2(ab+\overline{ab})$. If the commutator and the associator are written as $[a, b]=ab-ba$ and $(a, b, c)=(ab)c-a(bc)$ respectively, any inner derivation of $\mathfrak{A}$ can be generated by $D_{a,b}$ where $D_{a,b}(c)=[[a, b], c]-3(a, b, c)$.

In the composition algebra $\mathfrak{A}$, it is well known that the following identities hold (cf. R.D. Schafer [5]).

LEMMA 1.1. *For $a, b, c \in \mathfrak{A}$, we have that*

(1) $(ab, c)=(bc, a)$,

(2) $(a, b, c)=(b, c, a)=-(b, a, c)$,

(3) $D_{a,b}=-D_{b,a}$, $\overline{D_{a,b}(c)}=D_{\bar{a},\bar{b}}(\bar{c})$,

(4) $(Da, b)+(a, Db)=0$,

(5) $[D, D_{a,b}]=D_{Da,b}+D_{a,Db}$,

(6) $D_{ab,c}+D_{bc,a}+D_{ca,b}=0$,

where D is any inner derivation of $\mathfrak{A}$.

Let $M(n, \boldsymbol{R})$ denote an $n \times n$ matrix algebra over $\boldsymbol{R}$ with coefficients in $\boldsymbol{R}$. Let X, Y be elements in $M(n, \boldsymbol{R})$. We usually define a transposed operator

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