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ON THE RELATION BETWEEN PSEUDO-CONFORMAL AND KAHLER GEOMETRY

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Dedicated to Professor Shigeru Ishihara on his sixtieth birthday.

In [8, 9] S. Webster showed a relation between the geometry of a Kähler manifold and the pseudo-conformal geometry of a real hypersurface in $\boldsymbol{C}^{n+1}$. In particular the Bochner tensor of the Kähler manifold may be identified with the fourth order pseudo-conformal invariant of Chern and Moser [3] of a real hypersurface constructed to be a circle bundle over a neighbourhood of the Kähler manifold. It is this relation between the two geometries that we wish to study further here and we prove two results. The first is that if an infinitesimal pseudo-conformal transformation on the circle bundle of the Webster construction is projectable, the projected vector field is an infinitesimal isometry of the Kähler metric. The second theorem is that if a holomorphic transformation of a Kähler manifold preserves the Bochner tensor, its covariant derivatives and the tensor $D_{\alpha\beta}$ (see below), it is a homothety.

It is known that the Bochner tensor is a conformal invariant [7], but of course a conformal, non-homothetic change of a Kähler metric destroys the Kähler property. Though conceivable that there may be holomorphic transformations preserving the Bochner tensor other than homotheties we conjecture not and Theorem 2 is a result in this direction.

1. Pseudo-conformal geometry.

The problem of pseudo-conformal geometry is, given two real hypersurfaces of $\boldsymbol{C}^{n+1}$, can one find local differential invariants on them whose agreement is equivalent to the hypersurfaces being (locally) biholomorphically equivalent? This problem was solved by Chern and Moser [3] and the invariants are called pseudo-conformal or Chern-Moser invariants.

Consider a hypersurface M in $\boldsymbol{C}^{n+1}$ given by $r(z^1, \cdots, z^{n+1}, \bar{z}^1, \cdots, \bar{z}^{n+1})=0$ and such that for the real form $\theta=i\partial r$, the Levi form $d\theta$ is non-degenerate. In particular θ annihilates the holomorphic tangent space of M. We set $\mathcal{D}_p=\{X\in T_pM \mid \theta(X)=0\}$ and $\mathcal{H}_p=\{X-iJX \mid X\in\mathcal{D}_p\}$ where J is the almost complex structure on $\boldsymbol{C}^{n+1}$. Since J is integrable $[\mathcal{H}, \mathcal{H}]\subset\mathcal{H}$ and hence

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