A. ROS
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SPECTRAL GEOMETRY OF CR-MINIMAL SUBMANIFOLDS IN THE COMPLEX PROJECTIVE SPACE

BY ANTONIO ROS

Introduction. In the first part of this paper we will study an isometric imbedding of the complex projective space in the Euclidean space, see [7].

In the second part we use this imbedding and the total mean curvature theory, see [4], in order to obtain certain boundaries of the volume and the first eigenvalue of the spectrum of CR-minimal closed submanifolds of the complex projective space, such as certain characterizations of some of these submanifolds, in function of these geometric invariants. We give a λ_1-characterization of totally geodesic complex submanifolds, a spectral reduction of codimension theorem for totally real submanifolds and some other results.

Manifolds are assumed to be connected and dimension $n \geqq 2$ unless mentioned otherwise. For the necessary knowledge and notations of the geometry of submanifolds, see [2], and for spectral geometry, see [1].

1. An imbedding of the complex projective space in the Euclidean space.

Let $HM(n)=\{A \in gl(n, \boldsymbol{C}) / \bar{A}=A^t\}$ be the set of nxn-Hermitian matrices. $HM(n)$ is a n^2-dimensional linear subspace of $gl(n, \boldsymbol{C})$. We define in $HM(n)$ the metric

$$g(A, B)=2\,\mathrm{trace}\,(AB) \qquad \text{for all} \quad A, B \quad \text{in} \quad HM(n).$$

Let $\boldsymbol{C}P^n=\{A \in HM(n+1) / AA=A,\ \mathrm{trace}\, A=1\}$ and $U(n)$ be the unitary group.

LEMMA 1.1. *$\boldsymbol{C}P^n$ is a submanifold of $HM(n+1)$ diffeomorphic to $U(n+1)/U(1)\times U(n)$.*

Proof. Let A be in $\boldsymbol{C}P^n$. Since A is a Hermitian matrix, there exists P in $U(n+1)$ such that

$$PAP^{-1}=\begin{pmatrix} h_0 & & \\ & \ddots & \\ & & h_n \end{pmatrix}.$$

As $PAP^{-1}=(PAP^{-1})^2$, $h_i=h_i^2$, so that $h_i=0$ or $h_i=1$, but $\mathrm{trace}\,(PAP^{-1})=1$, therefore there exists an index i_0 such that $h_{i_0}=1$ and $h_i=0$ for all $i \neq i_0$.

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