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ON A CONJECTURE OF GACKSTATTER AND LAINE

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Dedicated to Professor Shigeru Ishihara on his 60th birthday

§1. **Introduction.** In their joint paper [2] Gackstatter and Laine offered the following conjecture: Let $a_i(z)$, $i=0, 1, \cdots, n-k$, be meromorphic in $|z|<\infty$. Then

$$w'^n=\sum_{i=0}^{n-k} a_i(z)w^i$$

does not admit any admissible solution, where $a_{n-k}(z)\not\equiv 0$ and k is an integer satisfying $1\leqq k\leqq n-1$. Here the admissible solution means a meromorphic solution of the given equation satisfying

$$T(r, a_i(z))=o(T(r, w))$$

for all i except for at most a set E of r of finite measure. In what follows this is simply denoted by $T(r, a_i)=S(r, w)$.

The above conjecture has a close connection with an unsolved problem due to Hayman ([4] Problem 1.21). Indeed, if the conjecture is true, then the simplest case $w'=a_0(z)$ implies that $T(r, w')=S(r, w)$ does not hold. However this is still unsolved, so far as the present author knows. Another simple case is $w'^n=a(w+\alpha)^{n-k}$ with a constant α and an integer $k(1\leqq k\leqq n-1)$. In this case

$$w+\alpha=\left(\frac{n}{k}\right)^{n/k}\left(\int^z a^{1/n}dz+C\right)^{n/k}.$$

Still we cannot decide whether $T(r, w')=S(r, w)$ or not.

In this paper we shall give a method to attack the above conjecture and prove the following

THEOREM. *Let $a_j(z)$, $j=0, 1, 2, 3$, be meromorphic in $|z|<\infty$. Then*

$$w'^n=a_3w^3+a_2w^2+a_1w+a_0, \qquad n\geqq 4, \quad a_3\not\equiv 0$$

does not admit any admissible solution with an exception of the following equation

$$w'^n=a_3(w+\alpha)^3$$

with a constant α.

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