ON SOLUTIONS OF A HOMOGENEOUS LINEAR MATRIX EQUATION WITH VARIABLE COMPONENTS

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1. We denote the totality of real numbers by ${\bf R}$ and the totality of complex numbers by ${\bf C}.$

Let I be a closed interval $[\alpha, \beta] = \{t \mid \alpha \leq t \leq \beta, t \in \mathbf{R}\}$. We denote by $C^{\mu}(I, \mathbf{R})$ the totality of real-valued functions defined and of class C^{μ} on I ($\mu = 0, 1, \dots, \infty$), and hereafter we fix some μ .

A complex-valued function f(t) defined on I is called a function of class C^{μ} on I if $\operatorname{Re} f(t) \in C^{\mu}(I, \mathbf{R})$ and $\operatorname{Im} f(t) \in C^{\mu}(I, \mathbf{R})$. We denote by $C^{\mu}(I, \mathbf{C})$ the totality of complex-valued functions defined and of class C^{μ} on I.

A d-dimensional row vector x with components $x_{\rho} \in \mathbb{C}$ $(\rho = 1, 2, \dots, d)$ will be denoted by

$$\mathbf{x} = (x_1, x_2, \dots, x_d)$$

and a d'-dimensional column vector y with components $y_{\sigma} \in \mathbb{C}$ ($\sigma = 1, 2, \dots, d'$) by

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{di} \end{pmatrix} = \operatorname{col}(y_1, y_2, \dots, y_{di}).$$

Now, let B(t) be a square matrix of degree n:

$$B(t) = \begin{pmatrix} b_{11}(t) & b_{12}(t) & \cdots & b_{1n}(t) \\ b_{21}(t) & b_{22}(t) & \cdots & b_{2n}(t) \\ \vdots & \vdots & & \vdots \\ b_{n1}(t) & b_{n2}(t) & \cdots & b_{nn}(t) \end{pmatrix},$$

where $b_{jk}(t) \in C^{\mu}(I, \mathbb{C})$ $(j, k=1, 2, \dots, n)$, and let us assume, throughout this paper, that for a positive integer $s: 1 \leq s \leq n-1$, a condition

(1)
$$\operatorname{rank} B(t) = n - s \ (=r)$$

is satisfied on the interval I, and further let us consider a homogeneous linear matrix equation

$$(2) B(t)P(t) = O,$$

where P(t) is an $n \times s$ matrix:

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