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ON SUBMANIFOLDS WITH FLAT NORMAL CONNECTION IN A CONFORMALLY FLAT SPACE

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1. Introduction.

In this paper we construct Gauss maps with respect to non-degenerate parallel normal unit vector fields on an n-dimensional submanifold N which has flat normal connection in an m-dimensional conformally flat space M $(2 \leqq n < m)$. A relation between the Riemannian curvatures of N, M and the Gauss images of N is obtained in theorem 1. We also find a result about the metric tensors of the Gauss images, which is in the case of a space form M closely related to a formula of Obata.

2. Preliminaries.

We always suppose that all manifolds, vector fields, etc. are differentiable of class C^{∞}. Assume that $\bar{\nabla}$ (resp. ∇) is the Riemannian connection of M (resp. N) and that X and Y are vector fields of N. Then

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y),$$

and h is the vector valued second fundamental tensor of N in M. Let ξ be a normal vector field on N. Decomposing $\bar{\nabla}_X \xi$ in a tangent and a normal component we find

$$\bar{\nabla}_X \xi = -A_{\xi}(X) + \nabla^{\perp}_X \xi .$$

A_{ξ} is a self-adjoint linear map $N_p \to N_p$ at each point p and $\nabla^{\perp}$ is a metric connection in the normal bundle $N^{\perp}$. We have also, if g denotes the metric tensor of M and the induced metric tensor on N,

$$g(h(X, Y), \xi) = g(A_{\xi}(X), Y) .$$

M is said to be conformally flat if for each point p we have a neighbourhood U and a diffeomorfism $\varphi : U \to R^m$, where R^m is the euclidean m-space, such that the metric tensor g of $\varphi(U)$ (identified with U) is obtained from the standard metric tensor of R^m by a conformal change of this tensor. Equivalently, g is locally of the form $g = \rho^2 g'$, where ρ is a strict positive function and g' is a flat metric tensor. The normal curvature tensor $R^{\perp}$ of N in M is given by

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