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A THEOREM ON THE SPREAD RELATION

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0. Introduction.

Let $u=u_1-u_2$ be nonconstant, where u_1 and u_2 are subharmonic in the plane **C**. For such a function u, we will write

$$N(r, u)=\frac{1}{2\pi}\int_{-\pi}^{+\pi}u(re^{i\theta})d\theta .$$

Then the Nevanlinna characteristic of $u=u_1-u_2$ is defined by

$$T(r)\equiv T(r, u)=N(r, u^+)+N(r, u_2).$$

For $b\in(-\infty, +\infty)$ we define

$$\sigma_b(r, u)=|\{\theta ; u(re^{i\theta})>b\}| .$$

(Here, and throughout this note, $|E|$ denotes the one-dimensional Lebesgue measure of the set E. Also, θ is understood to vary between $-\pi$ and $+\pi$.)

In [4], Baernstein proved the following result.

THEOREM A. *Suppose $u=u_1-u_2$ is nonconstant, where u_1 and u_2 are subharmonic in* **C**. *Let δ and λ be numbers satisfying*

$$\lambda>0, \qquad 0<\delta\leqq 1, \qquad \frac{4}{\lambda}\sin^{-1}\left(\frac{\delta}{2}\right)^{1/2}\leqq 2\pi .$$

Assume there exist $r_0\geqq 0$ and $b\in(-\infty, +\infty)$ such that $r\geqq r_0$ implies

$$N(r, u_2)\leqq(1-\delta)T(r, u)+O(1)$$

and

$$\sigma_b(r, u)<\frac{4}{\lambda}\sin^{-1}\left(\frac{\delta}{2}\right)^{1/2}. \tag{1}$$

Then

$$\lim_{r\to\infty}\frac{T(r, u)}{r^\lambda}=\alpha$$

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