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UNICITY THEOREMS FOR MEROMORPHIC OR ENTIRE FUNCTIONS, II

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0. Introduction. Let f and g be meromorphic functions. If f and g have the same a-points with the same multiplicities, we denote this by $f=a\rightleftarrows g=a$ for simplicity's sake. And we denote the order of f by ρ_f.

In [5] Ozawa proved the following result.

THEOREM A. *Let f and g be entire functions. Assume that $\rho_f, \rho_g<\infty$, $f=0\rightleftarrows g=0$, $f=1\rightleftarrows g=1$ and $\delta(0, f)>1/2$. Then $fg\equiv 1$ unless $f\equiv g$.*

It is natural to ask whether the order restriction of f and g in Theorem A can be removed or not. In our previous paper [6] we showed the following fact.

THEOREM B. *Let f and g be entire functions. Assume that $f=0\rightleftarrows g=0$, $f=1\rightleftarrows g=1$ and $\delta(0, f)>5/6$. Then $fg\equiv 1$ unless $f\equiv g$.*

In this paper we shall show first that in Theorem A the order restriction of f and g can be removed perfectly.

THEOREM 1. *Let f and g be entire functions. Assume that $f=0\rightleftarrows g=0$, $f=1\rightleftarrows g=1$ and $\delta(0, f)>1/2$. Then $fg\equiv 1$ unless $f\equiv g$.*

In Theorem 1, the estimate "$\delta(0, f)>1/2$" is best possible. In fact, consider $f=e^{\alpha}(1-e^{\alpha})$, $g=e^{-\alpha}(1-e^{-\alpha})$ with a nonconstant entire function α. Then $f=-ge^{3\alpha}$, $f-1=(g-1)e^{2\alpha}$ and $\delta(0, f)=1/2$. $f\not\equiv g$ and $fg\not\equiv 1$ are evident.

In place of Theorem 1, we prove more generally the following

THEOREM 2. *Let f and g be meromorphic functions satisfying $f=0\rightleftarrows g=0$, $f=1\rightleftarrows g=1$ and $f=\infty\rightleftarrows g=\infty$. If*

$$\varlimsup_{r\to\infty}\frac{N(r, 0, f)+N(r, \infty, f)}{T(r, f)}<1/2,$$

then $f\equiv g$ or $fg\equiv 1$.

In order to state our second result, we introduce a notation: If k is a positive integer or ∞, let

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