M. MAEDA
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# ROTATIONALLY INVARIANT CYLINDRICAL MEASURES I

BY MICHIE MAEDA

## § 0. Introduction.

Since Gross [4, 1962] introduced the concept of measurable norm, it has been extensively studied by many researchers (see for example [3, 4, 5, 6, 7]).

In a real separable Hilbert space $H$ there is a finitely additive cylindrical measure $\gamma$, say the canonical Gaussian cylindrical measure, which is analogous to the normal distribution in the finite dimensional case. Gross [5] showed that if $H$ is completed with respect to any of his measurable semi-norms, as defined in [4], then $\gamma$ gives rise to a countably additive Borel measure on the Banach space obtained from $H$ by means of the semi-norm. Dudley [2] showed that if the polar of the closed unit semi-ball is a compact $GC$-set, then the semi-norm is measurable in Gross' sense. Furthermore, using Dudley's result just above mentioned, Dudley-Feldman-Le Cam [3] proved the converse of Gross' result.

Here we shall generalize the above result for the rotationally invariant cylindrical measures. There are some inequalities for Gaussian cylindrical measures, known from Gross [4], which play an important role in the present circle of ideas. In this paper we shall begin to prove the similar inequalities concerning rotationally invariant cylindrical measures instead of Gaussian cylindrical measures.

On the other hand, Dudley-Feldman-Le Cam [3] introduced another measurability for semi-norms. They denoted Gross' definition by "*measurable by projections*" and the latter by "*measurable*". We shall use their expression. Badrikian-Chevet [1] have offered the problem whether these two concepts of measurability coincide exactly with each other. Our result will answer partially this problem.

Finally we add that this report contains [9] and improves the main result.

## § 1. Cylindrical measures.

Let $E$ be a real separable Banach space, $E^*$ its topological dual and $\mathcal{B}(E)$ the Borel $\sigma$-algebra of $E$. We use $(\cdot, \cdot)$ to denote the natural pairing between $E^*$ and $E$.

DEFINITION 1. Let $\{\xi_1, \cdots, \xi_n\}$ be a finite system of elements of $E^*$. Then by $\Xi$ we denote the operator : $x \in E \mapsto ((\xi_1, x), \cdots, (\xi_n, x)) \in R^n$. A set $Z \subset E$ is

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