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CRITICAL RIEMANNIAN METRICS ON SASAKIAN MANIFOLDS

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1. Introduction. Let g be a Riemannian metric which is defined on a compact orientable differentiable manifold M of dimension n and makes its volume V_g equal to 1, that is, $\int_M dV_g=1$, where dV_g is the volume element of M measured by g. We denote the set of all such metrics by $\mathfrak{M}$. When g is fixed we have a Riemannian manifold (M, g). Let us take a covering $\{U\}$ of M by coordinate neighborhoods and denote the local coordinates in U by $\{x^a\}$, where $a, b, c, \cdots$ run over the range $\{1, 2, 3, \cdots, n\}$. In each U, g is expressed by its components g_{ab}. We adopt summation convention so that the contravariant components g^{ab} of g satisfy $g_{ac}g^{bc}=\delta_a{}^b$. By $R_{abc}{}^d$, R_{ab} and R we denote the components of the Riemannian curvature tensor, the Ricci tensor and the scalar curvature of (M, g), respectively. Now let us consider the integral

$$F_M[g]=\int_M f(R)dV_g\,,$$

where $f(R)$ is a scalar field on M determined by g as the contraction of a tensor product of the curvature tensor. This integral defines a mapping $F:\mathfrak{M}\rightarrow R$. A critical point of F is denoted by g_F and is called a critical Riemannian metric with respect to the field $f(R)$ or the integral $F_M[g]$. The following four kinds of critical Riemannian metrics have been studied by M. Berger [1] and Y. Mutō [5, 6, 7, 8, 9]:

$$A_M[g]=\int_M R\,dV_g\,, \qquad B_M[g]=\int_M R^2\,dV_g\,,$$

$$C_M[g]=\int_M R_{ab}R^{ab}\,dV_g\,, \quad D_M[g]=\int_M R_{abcd}R^{abcd}\,dV_g\,.$$

The equations of the critical Riemannian metric are written as follows:

$$(1.1)\qquad A_{ab}=C_Ag_{ab}\,,\quad B_{ab}=C_Bg_{ab}\,,\quad C_{ab}=C_Cg_{ab}\,,\quad D_{ab}=C_Dg_{ab}\,,$$

where C_A, C_B, C_C and C_D are undetermined constants and A_{ab}, B_{ab}, C_{ab} and D_{ab} are given by

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